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COMPARISON OF HIGHER-ORDER AMBISONICS AND WAVE FIELD SYNTHESIS WITH RESPECT TO SPATIAL ALIASING ARTIFACTS

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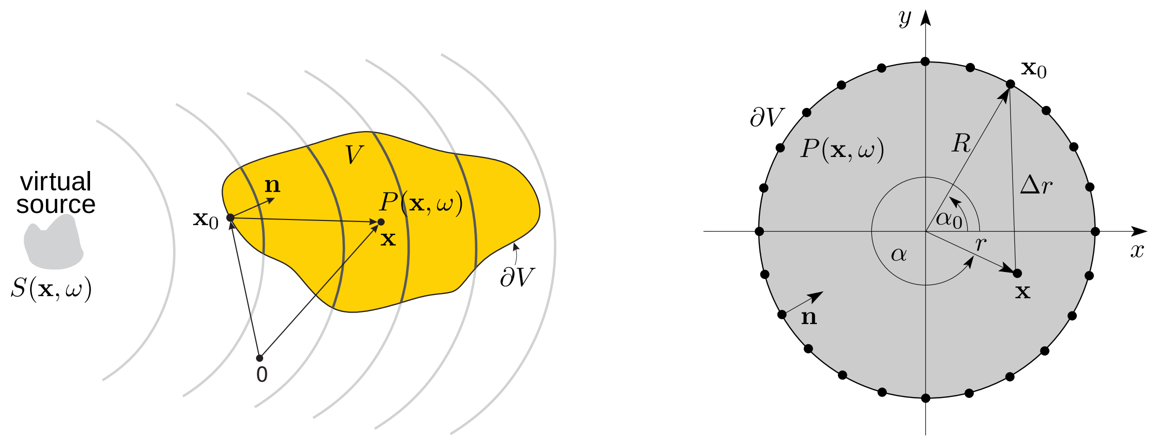
ABSTRACT
Wave field synthesis (WFS) and higher-order Ambisonics (HOA) are two sound reproduction techniques that facilitate a high number of reproduction channels to overcome the sweet-spot limitation known from conventional stereophonic reproduction techniques. HOA is based on a representation of the reproduced wave field in terms of spatial harmonics, WFS is based on the Kirchhoff-Helmholtz integral. It has been shown that HOA and WFS are comparable when applying a near-field correction to HOA and assuming an infinite number of loudspeakers around the listening area. However, both methods diverge with respect to their spatial aliasing properties for a finite number of loudspeakers. This contribution investigates the difference of HOA and WFS in terms of spatial aliasing artifacts for two-dimensional reproduction systems.

INTRODUCTION
Various massive multichannel reproduction systems have been proposed to enable spatial rendering of sound. The best-known of which are wave field synthesis (WFS) and higher-order Ambisonics (HOA). Although both of them are derived on the basis of the wave equation, the approaches come from different directions and their relationship is not clear.
The work presented in [3, 5] has revealed a strong connection between WFS and HOA under certain conditions. These conditions are: (1) an infinite number of loudspeakers and (2) a near-field correction for HOA. However, a practical system will only have a limited number of loudspeakers. This might result in spatial aliasing artifacts present in the reproduced wave field. It was already noted in [3] that WFS and HOA show different performance w.r.t. spatial aliasing. Here, we will extend the comparison of WFS and HOA to the behavior of HOA and WFS in terms of their spatial aliasing properties.
This paper is organized as follows: We first introduce the theoretical basics of WFS and HOA. We then apply specializations that allow for a comparison of both reproduction techniques. For WFS this specialization is a circular distribution of secondary sources and sampling of the secondary source distribution, for HOA it is a near-field compensation. We will then dedicate a section to the emphasis of the similarities and differences of WFS and HOA with respect to spatial sampling artifacts and illustrate the observations with numerical simulations.

Nomenclature
We will restrict the descriptions in this paper to two dimensional reproduction. Two dimensional in this context means that an observed sound field is independent from one of the spatial coordinates, e.g. $P(x, y, z, \omega) = P(x, y, \omega)$. The following conventions are used: For scalar variables lower case denotes the time domain, upper case the temporal frequency domain. Vectors are denoted by lower case boldface. The Cartesian and polar coordinate system are used. The



(a) arbitrary secondary source contour. (b) (sampled) circular secondary source contour.

Figure 1: Geometry used to derive the wave fields reproduced by WFS and HOA. The dots • denote the spatial sampling positions.

two-dimensional position vector in Cartesian coordinates is given as $\mathbf{x} = [x\ y]^T$. The Cartesian coordinates are linked to the polar coordinates by $x = r\cos\alpha$ and $y = r\sin\alpha$. The acoustic wavenumber is denoted by k. It is related to the temporal frequency by $k = \left|\frac{\omega}{c}\right|$ with ω being the radial frequency and c the speed of sound.

WAVE FIELD SYNTHESIS

WFS is a spatial sound reproduction technique which is essentially based on the Kirchhoff-Helmholtz integral [1]. This fundamental principle states that a continuous distribution of appropriately driven secondary monopole and dipole sources placed around the desired listening area is suitable to reproduce any virtual wave field. Several simplifications are necessary to arrive at a realizable two-dimensional sound reproduction system. One of these is to overcome the need of secondary dipole sources. The wave field reproduced by an arbitrarily shaped secondary source contour ,consisting of monopoles only, is given as [6]

$$P(\mathbf{x},\omega) = -\oint_{\partial V} \underbrace{2a(\mathbf{x_0})\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega)}_{D_{\text{WFS}}(\mathbf{x}_0,\omega)} G(\mathbf{x}|\mathbf{x}_0,\omega)\, dS_0 \ , \qquad \text{(Eq. 1)}$$

where $G(\mathbf{x}|\mathbf{x}_0,\omega)$ denotes a free-field Green's function, $\partial/\partial\mathbf{n}$ the directional gradient, $\mathbf{x}$ a point within the listening area V, $\mathbf{x}_0$ a point on the boundary ∂V of V, $S(\mathbf{x},\omega)$ the field of the virtual source, $a(\mathbf{x_0})$ a window function which takes care that only the relevant secondary sources are excited [8] and $D_{\text{WFS}}(\mathbf{x}_0,\omega)$ the secondary source driving function. The geometry is illustrated in Fig. 1(a). For a two-dimensional reproduction scenario the Green's function is given as $G(\mathbf{x}|\mathbf{x}_0,\omega) = \frac{j}{4}H_0^{(2)}(k\,|\mathbf{x}-\mathbf{x}_0|)$, which can be interpreted as the field of a (monopole) line source. Please note that two-dimensional WFS systems typically use point sources as secondary sources, which results in amplitude errors in the reproduced wave field [6]. However, these artifacts do not play a role for the analysis of aliasing artifacts for WFS and HOA.
For a comparison of WFS with HOA the wave field reproduced by a circularly shaped secondary source distribution with radius R is investigated. Figure 1(b) illustrates the geometry. The reproduced wave field $P(\mathbf{x},\omega)$ inside the secondary source distribution ($r < R$) is given by specializing Eq. 1 to the circular geometry [10]

$$\begin{aligned} P(\mathbf{x},\omega) &= -\frac{j}{4}\int_0^{2\pi} D_{\text{WFS}}(\alpha_0,R,\omega)\, H_0^{(2)}(k\Delta r)\, R\, d\alpha_0 \\ &= -j\frac{\pi}{2}R \sum_{\nu=-\infty}^{\infty} J_\nu(kr)\, H_\nu^{(2)}(kR)\, \mathring{D}_{\text{WFS}}(\nu,R,\omega)\, e^{j\nu\alpha} \ , \end{aligned} \qquad \text{(Eq. 2)}$$

where $\Delta r = |\mathbf{x} - \mathbf{x}_0|$ and $\mathring{D}_{\text{WFS}}(\nu, R, \omega)$ are the Fourier series coefficients of the driving function. For the derivation of the second equality the definition of the Fourier series and the addition theorem for Hankel functions [4] are used to eliminate the angular integral. Eq. 2 states that the reproduced wave field is given by a Fourier series with respect to the angle α. The summation index ν can be interpreted as angular frequency. The coefficients of the series are given by $\mathring{D}_{\text{WFS}}(\nu, R, \omega)$ weighted by a Bessel and a Hankel function.

Since arbitrary wave fields can be decomposed into plane waves [11] it is sufficient to consider a plane wave as virtual source wave field to analyze the aliasing artifacts of WFS and HOA.

The virtual source field for a Dirac shaped plane wave with propagation direction θ_{pw} is given as follows

$$S_{\text{pw}}(\mathbf{x}, \omega) = e^{-jkr\cos(\theta_{\text{pw}} - \alpha)} = \sum_{\nu=-\infty}^{\infty} j^{-\nu} J_\nu(kr) e^{-j\nu\theta_{\text{pw}}} \, e^{j\nu\alpha} \;, \tag{Eq. 3}$$

where the Fourier series representation is known as Jacobi-Anger expansion. As indicated in Eq. 1 the driving function for a plane wave can be derived by calculating the directional gradient of $S_{\text{pw}}(\mathbf{x}, \omega)$, evaluating it at $r = R$ and taking only the incoming contributions of the wave field as

$$D_{\text{WFS}}(\alpha_0, R, \omega) = -2k \sum_{\nu=-\infty}^{\infty} j^{-\nu} H_\nu^{'}(kR) e^{-j\nu\theta_{\text{pw}}} \, e^{j\nu\alpha_0} \;, \tag{Eq. 4}$$

where $H_\nu^{'}(\cdot)$ denotes the derivative of the Hankel function with respect to its argument. Up to now a continuous distribution of secondary sources was assumed. In practical systems this distribution will be discrete.

Sampling of circular secondary source distribution

The discretization the secondary source distribution is modeled by sampling the loudspeaker driving function $D_{\text{WFS}}(\alpha_0, R, \omega)$ at N equidistant angles (see Fig. 1(b)). Angular sampling results in repetitions of the angular spectrum [10]. The Fourier series coefficients $\mathring{D}_{\text{S,WFS}}(\nu, R, \omega)$ of the sampled driving function are given as

$$\mathring{D}_{\text{S,WFS}}(\nu, R, \omega) = \sum_{\eta=-\infty}^{\infty} \mathring{D}_{\text{WFS}}(\nu + \eta N, R, \omega) \;. \tag{Eq. 5}$$

Introducing Eq. 5 into Eq. 2 describes the wave field reproduced by a discrete secondary source distribution.

The reproduction process for a discrete distribution of secondary sources, as described above, can be understood as a sampling and interpolation process. The driving function is sampled at the secondary source positions and interpolated by the secondary sources into the listening area. In this sense alias-free reproduction for a circular secondary source distribution is only possible if (1) the driving function and (2) the secondary sources are band-limited in the angular frequency domain. It can be seen from Eq. 4 that this is not the case for the reproduction of a plane wave, due to the properties of the Hankel functions. Without further modification aliasing is present in the sampled driving function $\mathring{D}_{\text{S,WFS}}(\nu, R, \omega)$. Aliasing could be reduced by limiting the angular bandwidth of the driving function prior to the sampling process

$$\mathring{D}_{\text{WFS}}(\nu, R, \omega) = \begin{cases} \mathring{D}_{\text{WFS}}(\nu, R, \omega) & \text{for } -N' \leq \nu \leq N', \\ 0 & \text{otherwise.} \end{cases} \tag{Eq. 6}$$

where $N' = (N-1)/2$ for odd N. However, this will not result in an alias-free reproduction since the spectrum of the secondary sources is not band-limited in the angular frequency domain as can be seen from Eq. 2. Please note that the angular bandwidth of the plane wave driving function is currently not limited in most circular WFS implementations, since the driving function is typically calculated in the time-domain. The only possibility for alias-free reproduction would be to modify the angular spectrum of the secondary sources in the angular frequency domain.

HIGHER-ORDER AMBISONICS

The Ambisonics principle is based on the expansion of the desired sound field and the sound fields of the secondary sources into spatial harmonics. The loudspeaker driving signals are chosen such that the superposition of their sound fields approximates the desired one. It is typically assumed that the secondary sources are located on a sphere around the listener residing in the center. In the two-dimensional reproduction scenario considered here, the secondary sources are located on a circle. We assume an evenly spaced distribution of N loudspeakers as depicted in Fig. 1(b). The reproduced wave field is then given as

$$P(\mathbf{x},\omega) = \sum_{n=0}^{N-1} D_{\text{HOA}}(\alpha_n, R, \omega) V(\mathbf{x}, \mathbf{x}_n, \omega) \; , \tag{Eq. 7}$$

where $D_{\text{HOA}}(\alpha_n, R, \omega)$ denotes the driving signal for the n-th secondary source situated at $\mathbf{x}_n = R \cdot [\cos\alpha_n \;\; \sin\alpha_n]^T$. The driving signals are typically derived from microphone recording techniques which utilize a Fourier series representation of the recorded signals. The recording step is referred to as *encoding*, the rendering as *decoding* in the context of Ambisonics. As stated before, we are interested in the reproduction of virtual plane waves. For ease of illustration we will combine the en-/deconding step in order to derive the driving signal for HOA in closed form. The driving signal $D_{\text{HOA}}(\alpha_n, R, \omega)$ for HOA is derived by setting the left hand side of Eq. 7 to the desired wave field and solving the resulting equation w.r.t. the driving function. Due to the underlying circular geometry a Fourier series representation of the wave fields is convenient. As for WFS the desired wave field is a plane wave with propagation direction θ_{pw} for the following derivation of driving functions.

Traditional amplitude panning approach

It is assumed for the traditional amplitude panning approach of HOA that the secondary sources are far enough away from the origin so that their wave fields there can be approximated by plane waves. Introducing $S_{\text{pw}}(\mathbf{x},\omega)$ as desired wave field and accordingly plane waves with propagation direction $\theta_n = \alpha_n + \pi = n\frac{2\pi}{N}$ as secondary source fields into Eq. 7 yields

$$e^{-jkr\cos(\theta_{\text{pw}}-\alpha)} = \sum_{n=0}^{N-1} D_{\text{HOA}}(\alpha_n, R, \omega) e^{-jkr\cos(\theta_n-\alpha)} \; . \tag{Eq. 8}$$

Expanding the exponential functions on both sides of Eq. 8 into a Fourier series with respect to the angle α using the Jacobi-Anger expansion and comparing the coefficients of these series results in

$$e^{-j\nu\theta_{\text{pw}}} = \sum_{n=0}^{N-1} D_{\text{HOA}}(\alpha_n, R, \omega) \; e^{-j\nu n\frac{2\pi}{N}} \; . \tag{Eq. 9}$$

The angular frequency ν is also referred to as *mode* in this context and the approach consequently as *mode matching*. Typically Eq. 9 is solved w.r.t. the driving functions by interpreting it as a set of linear equations for the angular frequencies $-N' < \nu < N'$. The problem can be formulated conveniently in a matrix formulation. However, in order to find a closed-form solution we will chose a different approach here [7]. The sum on the right hand side of Eq. 9 can be interpreted as a discrete Fourier transform (DFT) of length N w.r.t. the variable n. Hence, the right hand side represents the DFT of the driving signal $D_{\text{HOA}}(\alpha_n, R, \omega)$. Consequently Eq. 9 can be solved by applying an inverse DFT (IDFT) to both sides

$$D_{\text{HOA}}(\alpha_n, R, \omega) = \frac{1}{N} \sum_{\nu=-N'}^{N'} e^{-j\nu\theta_{pw}} e^{j\nu\theta_n} = \frac{\sin(N(\alpha_n + \pi - \theta_{\text{pw}})/2)}{N\sin((\alpha_n + \pi - \theta_{\text{pw}})/2)} \; . \tag{Eq. 10}$$

For the traditional approach the driving function only weights the individual secondary sources with the amplitude gains given by Eq. 10. The assumption made above that the secondary sources approximately have the characteristics of plane waves is not required for WFS. In order to compare both reproduction techniques we will illustrate the extension of the traditional panning approach by incorporating line sources as secondary sources. This approach is typically referred to as *near-field compensated HOA* [2].

Near-field compensation

The near-field compensated driving signals are derived by setting $V(\mathbf{x}, \mathbf{x}_n, \omega) = \frac{j}{4} H_0^{(2)}(k\,|\mathbf{x} - \mathbf{x}_n|)$ instead of using the plane wave assumption for the secondary source fields, then applying the addition-theorem of the Hankel function, and performing the same steps as outlined for the panning approach. The near-field compensated driving functions are then given as

$$D_{\mathsf{HOA}}(\alpha_n, R, \omega) = \frac{4}{j}\frac{1}{N}\sum_{\nu=-N'}^{N'} j^{-\nu}\frac{1}{H_\nu^{(2)}(kR)} e^{-j\nu\theta_{\mathsf{pw}}}\, e^{j\nu\theta_n} \qquad \text{(Eq. 11)}$$

The limitation of the angular frequencies ν in the mode matching approaches outlined above results in a finite series expansion of the plane wave on the left hand side of Eq. 7. Due to the properties of the Bessel functions, the approximation of the plane wave will be better for low frequencies and smaller distances to origin [3]. For very low orders the wave field will only be exact at the center of the array, hence the listening area will be very limited.

COMPARISON OF WFS AND HOA

For a comparison of WFS and near-field corrected HOA we will first take a closer look at their reproduced wave fields, as given by Eq. 1 and Eq. 7. Introducing the Green's function into Eq. 7 and applying the addition-theorem of the Hankel functions yields that the reproduced wave fields are (up to a scalar factor) equal for $N \rightarrow \infty$ when using the same driving function for WFS and HOA. However, the method used to calculate the driving function differs between both. WFS relies on a derivation of the driving function on basis of the Kirchhoff-Helmholtz integral, HOA on comparing the reproduced wave field to the desired wave field. For two-dimensional reproduction WFS may have near-field artifacts [9] which might be the reason why the driving functions of WFS and HOA differ. Of special interest in the scope of this paper is the difference in terms of spatial aliasing.

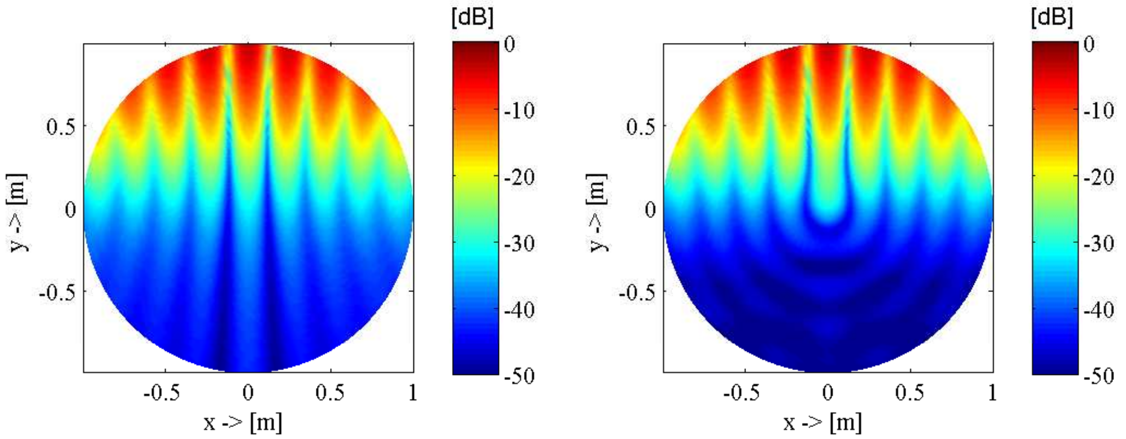
WFS is based on a description of the driving function in continuous space which is then discretized for practical realization. For the reproduction of plane waves on circular secondary source contours this will result in spatial aliasing as outlined in this paper. These artifacts can be reduced to some extend by limiting the angular bandwidth of the driving function. The limitation of the angular bandwidth of a plane wave is inherently included in the mode-matching approach used for HOA. However due to the properties of the DFT, the angular spectrum of the driving function for HOA will also exhibit spectral replications of the bandlimited angular spectrum of a plane wave. Hence, regarding spatial aliasing WFS and HOA are similar when limiting the angular bandwidth of the plane wave for calculation of the WFS driving function. Typically this limitation is not performed in practical WFS systems and as a result WFS and HOA differ w.r.t. their spatial aliasing characteristics.

In order to illustrate the effect of bandwidth limitation we computed the reproduced aliasing-to-signal ratio (RASR) for a plane wave which is not-bandlimited and band-limited in the angular frequency domain. The RASR is defined as the energy ratio of the reproduced aliasing contributions to the desired wave field [10] with varying (temporal) bandwidth of the reproduced wave field. In general, the RASR will depend on the desired wave field and the listener position. The RASR is zero ($-\infty$ dB) for alias-free reproduction.

Figure 2 illustrates the RASR for a WFS system with $N = 56$ secondary line sources placed on a circular contour with a radius of $R = 1.5$ m. The geometrical parameters are chosen in accordance to the WFS system installed at the Usability Laboratory of the Deutsche Telekom Laboratories. The wave field used for the virtual source is a Dirac shaped plane wave bandlimited to $f_{\mathsf{max}} = 750$ Hz with propagation direction $\theta_{\mathsf{pw}} = \frac{3\pi}{2}$. Figure 2(a) shows the RASR when not applying a bandlimitation in the angular frequency domain, as this is the typical case for WFS systems. Figure 2(a) shows the RASR when applying a bandlimitation in accordance to the concept of HOA. It can be seen clearly that the spatial aliasing artifacts differ for both cases. For the bandlimited case the aliasing properties are slightly better.

CONCLUSIONS

This paper presents a specialization of the theoretical basics of WFS and HOA that allows for an analytical comparison of both techniques. For the specialized situation considered here, i.e. a Dirac shaped plane wave rendered by a circular distribution of evenly spaced secondary monopole line sources, the major difference between HOA and WFS with respect to spatial aliasing is the fact that in HOA the loudspeaker driving function is inherently bandlimited with respect to the angular spectrum, thus preventing some amount of spatial aliasing. This is not the case for WFS.



(a) without limitation of the angular bandwidth.

(b) with limitation of the angular bandwidth.

Figure 2: RASR for a discrete circular secondary source distribution with $N = 56$ positions and a radius of $R = 1.50$ m when reproducing Dirac shaped plane wave bandlimited to $f_{\text{max}} = 750$ Hz.

Both techniques perform equal if the angular spectrum of the driving function for WFS is also bandlimited. However, even then the reproduced wave field is not free of spatial aliasing artifacts. Further research will include an extension of the presented analysis to near-field effects.

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