Evaluation of the Circular Harmonics Decomposition for WDAF-based Active Listening Room Compensation

Sascha Spors[1] and Rudolf Rabenstein[2]

[1]*Deutsche Telekom Laboratories, Technical University of Berlin, Ernst-Reuter Platz 7, 10587 Berlin, Germany*

[2]*Telecommunications Laboratory, University of Erlangen-Nuremberg, Cauerstrasse 7, 91058 Erlangen, Germany*

Correspondence should be addressed to Sascha Spors (`Sascha.Spors@telekom.de`)

ABSTRACT

The underlying theory of most massive multichannel spatial sound reproduction systems assumes ideal, e. g. free-field propagation characteristics in the listening room. The effects of non-ideal listening rooms can be reduced by active listening room compensation. A very efficient realization of such a system is given by wave-domain adaptive filtering. Here spatio-temporal signal transformations are used to transform the adaptation problem into a highly reduced representation. However, the optimal transformation depends on the acoustic properties of the listening room, which are in general not known a-priori. Practical implementations therefore use listening room independent transformations. This paper evaluates the circular harmonics decomposition as wave domain transformation. Its favorable properties are demonstrated by simulations and laboratory measurements.

1. INTRODUCTION

The reproduction of spatial audio to large audiences requires methods that go beyond conventional surround sound systems with 5 or 7 channels. A true reproduction of recorded or virtual acoustic scenes in an extended area requires several ten to more than hundred loudspeakers. These systems are called massive multichannel spatial sound reproductions systems. Frequently used approaches are wave field synthesis (WFS) [1, 2] or (higher-order) Ambisonics [3, 4]. Both approaches are based on different mathematical representations of wave propagation as given by the acoustic wave equation. However, these representations assume free field conditions and are only valid in open space or in reflection free enclosures.

In practice, sound reproduction systems are located in enclosures with non-negligible reflections. These reflections can be reduced to some extent by passive damping methods, but they cannot be totally eliminated if the enclosure shall remain an attractive listening environment for the audience. Such an enclosure is called the listening room.

Since massive multichannel reproduction systems provide a large number of independent loudspeaker positions, it is feasible to use this system not only for spatial reproduction but also for the active compensation of unwanted reflections at the walls of the listening room. A compensation effect can be achieved by appropriate pre-equalization of the individual driving signals for each loudspeaker. This approach is called *active listening room compensation*.

It is not an easy task, since an exact cancelation of reflections requires the exact knowledge ot the room response at all times. Temperature variations, opening or closing of doors, movement of persons in the audience, and other influences produce constant changes to the room response. Therefore active listening room compensation requires to record the current state of the room response by an appropriate configuration of microphones. Monitoring the listening room response in such a way is called wave field analysis (WFA). In very general terms, a massive multichannel reproduction system with active listening room compensation consists of (1) an array of loudspeakers, (2) an array of microphones, (3) an adaptive system for the computation of the pre-equalized loudspeaker driving signals.

The mathematical basis of the adaptive system is the topic of this contribution. With some ten or more loudspeakers and an equal number of microphones, the resulting multiple-input/multiple-output (MIMO) adaptive system has paths in the order of thousands. Consequently, traditional adaptation approaches are bound to fail. A solution is to decouple the adaptation through a transition into the so-called wave domain (*wave domain adaptive filtering, WDAF*). This transition requires the choice of a certain mathematical basis for the representation of the signals. This contribution evaluates the suitability of the *circular harmonics decomposition* for this purpose.

Section 2 presents the aspects of active listening room compensation mentioned above in more detail. Section 3 introduces performance measures for WDAF. The core results of this contribution are given in Section 4, where the circular harmonics decomposition is evaluated in the context of WDAF.

2. ACTIVE LISTENING ROOM COMPENSATION

A first approach to active listening room compensation would be to analyze the reproduced sound field in the listening area with microphones and to infer compensation signals from the microphone signals. Besides the high number of loudspeakers used for massive multichannel reproduction systems, an adequate analysis of the listening room influence also requires a quite high number of reference microphones. Since the acoustic properties of the listening room may change over time, the compensation signals have to be computed using adaptive algorithms. Due to the high number of analysis and synthesis signals, an adaptation of the compensation signals with traditional algorithms is subject to fundamental problems [5].

Recently a novel approach to adaptive filtering for massive multichannel acoustic systems, wave domain adaptive filtering (WDAF), has been proposed [6, 7, 8]. This approach explicitly considers the wave nature of sound to overcome the fundamental problems of traditional adaptation algorithms within this context. The particular implementation [6] of the active listening room compensation system considered in the sequel uses WFS for reproduction, WFA techniques for the analysis of the reproduced wave field and WDAF for an efficient adaptation of the compensation signals.

The next sections will briefly introduce the concepts of WFS, WFA and WDAF, and will propose an active room compensation system for typical listening rooms.

2.1. Wave Field Synthesis

Wave field synthesis aims at reproducing the sound of complex acoustic scenes as natural as possible. In contrast to other multi-channel approaches, it is based on fundamental acoustic principles. WFS allows a physically correct reproduction of wave fields. In doing so, WFS overcomes the limitations of the traditional multichannel systems like the “sweet-spot”. This section gives a brief introduction into the basics of WFS.

The theory of WFS has been initially developed at the Technical University of Delft [9]. The intuitive foundation of WFS is given by Huygens’ principle. Huygens stated that any point of a propagating wave front at any time-instant conforms to the envelope of spherical waves emanating from every point on the wavefront at the prior instant. This principle can be utilized to synthesize acoustic wavefronts of arbitrary shape. WFS has also a solid physical basis besides this more illustrative description. The mathematical foundation of Huygens’ principle is given by the Kirchhoff-Helmholtz integral [10].

In order to arrive at a practical implementation several simplifications with respect to the theory have to be performed. These may lead to artifacts in the reproduced wave field, as discussed in detail by [11, 12, 13]. From the artifacts of WFS only spatial aliasing will be considered in the following.

Spatial aliasing artifacts emerge from the fact that a practical implementation of a WFS system can only be realized by a limited number of loudspeakers placed at discrete positions. Spatial aliasing limits the frequency up to which a proper control is gained over the wave field. Since active room compensation is built upon destructive interference, its application is limited by spatial aliasing. The spatial aliasing artifacts and anti-aliasing conditions for linear/circular loudspeaker arrays have been discussed by [1, 11, 14]. In the following, it is assumed that the spatial aliasing conditions are reasonable fulfilled by the particular reproduction system and scenario employed.

2.2. Wave Field Representations and Analysis

Active room compensation requires to analyze the reproduced wave field. In principle, microphones placed at well chosen positions within the listening area could be used for the purpose. However, it is much more elaborate to use wave field analysis techniques to analyze and represent the reproduced wave field. The following discussion will be limited to wave field representations and analysis for two-dimensional wave fields as this is suitable for typical sound reproduction systems.

A wave field can be decomposed into the free-field eigensolutions of the wave equation. These eigensolutions depend on the particular coordinate system used. Common choices for coordinate systems in two-dimensional space are the Cartesian and polar coordinate system. The representations of a wave field that are connected to Cartesian and polar coordinates decompose a wave field into plane waves and circular harmonics, respectively [15]. The decomposition of an acoustic field into plane waves decomposes an arbitrary wave field into a set of plane waves from all possible directions [15, 16].
Of more interest within the scope of this paper is the decomposition of an acoustic wave field into circular harmonics. This decomposition is given as follows [15]

$$P(\alpha,r,\omega) = \sum_{\nu=-\infty}^{\infty} \breve{P}^{(1)}(\nu,\omega) H_\nu^{(1)}(\frac{\omega}{c}r) e^{j\nu\alpha} + \sum_{\nu=-\infty}^{\infty} \breve{P}^{(2)}(\nu,\omega) H_\nu^{(2)}(\frac{\omega}{c}r) e^{j\nu\alpha} , \tag{1}$$

where $\breve{P}^{(1),(2)}(\nu,\omega)$ denote the expansion coefficients in terms of circular harmonics, $H_\nu^{(1),(2)}(\cdot)$ the Hankel function of ν-th order of first/second kind, ω and ν the temporal and angular frequency , r and α the radial and angular coordinate of the polar coordinate system and c the speed of sound. It can be shown that $\breve{P}^{(1)}(\nu,\omega)$ represents the expansion of an incoming wave field with respect to the origin and and $\breve{P}^{(2)}(\nu,\omega)$ the expansion of an outgoing wave field [15]. The decomposition into incoming and outgoing waves can be used to distinguish between sources inside and outside the measured area. The expansion coefficients in terms of circular harmonics $\breve{P}^{(1),(2)}(\nu,\omega)$ and plane waves $\bar{P}^{(1),(2)}(\theta,\omega)$ are related to each other by a Fourier series [16].
Note that the evaluation of Eq. (1) requires access to the entire two-dimensional wave field $P(\alpha,r,\omega)$ to compute the expansion coefficients. However, the Kirchhoff-Helmholtz integral states that measurements taken at the boundary of the region of interest are suitable to characterize the wave field within that region. In a practical implementation these measurements can only be taken at discrete points on the boundary. This may result in spatial aliasing if the sampling is not performed properly. Using the decomposition into circular harmonics for circular microphone arrays provides an efficient wave field analysis tool, as illustrated by [16]. It will be assumed in the following that a circular microphone array is deployed for the analysis of the reproduced wave field, since circular microphone arrays have many desirable properties (e. g. constant characteristics with respect to the angle of incident plane waves).

2.3. Active Listening Room Compensation using Wave Domain Adaptive Filtering

Active compensation of the listening room acoustics can be performed by pre-equalization of the loudspeaker driving signals with suitable compensation filters. The adaptation of these filters by adaptive algorithms is subject to fundamental problems for massive multichannel reproduction systems [5]. In the following, we will shortly review the room compensation system for WFS that was proposed in [6]. It is based upon the concept of wave domain adaptive filtering (WDAF).
Instead of using the loudspeaker and microphone signals directly to calculate room compensation filters a spatio-temporal transformation of the respective signals into a different representation is performed. This representation is termed as *wave domain* in the following. Optimally, this transformation entirely decouples the MIMO adaptation problem into a series of single-channel adaptation problems. The computational complexity of adapting the room compensation filters is reduced significantly this way. Figure 1 shows a generic block diagram of the proposed room compensation system. The transfer functions from each loudspeaker to each microphone position are combined in the listening room response $\underline{\mathbf{R}}(\omega)$. Transformations $\mathscr{T}_2$ and $\mathscr{T}_3$ are designed such that they jointly decouple the listening room $\underline{\mathbf{R}}(\omega)$ and free-field response $\underline{\mathbf{F}}(\omega)$. As a consequence, the matrix of compensation filters $\underline{\breve{\mathbf{C}}}(\omega)$ can be decomposed into a set of compensation filters, each acting on only one spatial signal component (diagonal matrix). The adaption of these compensation filters is then performed independently for each spatially transformed component. The number of compensation filters that have to be adapted is lowered significantly compared to the traditional approaches on the one hand and the spatial correlations are removed on the other hand. Thus, the complexity of the filter adaptation is highly reduced and the numerical conditioning of the problem is improved. Please note, that nearly all one channel filtered-x adaptation algorithms can be utilized to adapt the compensation filters in the transformed domain. The transformation $\mathscr{T}_1$ transforms the virtual source $Q(\omega)$ to be auralized into its wave domain representation. Suitable spatial source models, like point source or plane wave propagation, can be prescribed for the spatial characteristics of the source. However, it is also possible to prescribe complex wave fields as desired

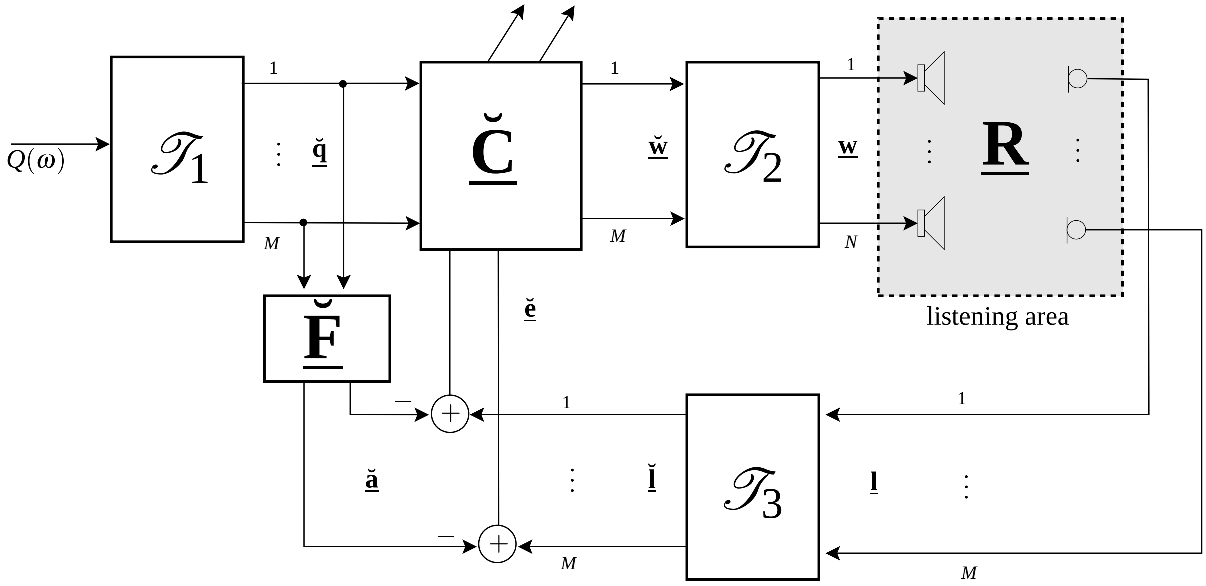


Fig. 1: Block diagram of a room compensation system based on eigenspace/wave domain adaptive filtering. Vectors are denoted by lower case boldface, matrices by upper case boldface variables. The temporal frequency domain for vectors and matrices is denoted by underlining the respective variables. The wave domain is denoted by a wedge placed over the symbol.

wave field.

The optimal wave domain representation is given by a generalized singular value decomposition (GSVD) of the listening room $\underline{\mathbf{R}}(\omega)$ and free-field transfer $\underline{\mathbf{F}}(\omega)$ matrices [5]. Since the listening room transfer matrix $\underline{\mathbf{R}}(\omega)$ will depend on the acoustic characteristics of the listening room, the optimal transformation will also. Since the characteristics of the listening room are not known a-priori or may change over time this optimal solution is of more theoretical interest.

A practical solution can be derived by releasing the requirement for an optimal decoupling of the adaptive system. The basic idea of WDAF is to use fixed transformations that approximately decouple the adaptive system.

2.4. Circular Harmonics as Wave Domain Transformation

The optimal choice for the transformed signal representation depends on the acoustic characteristics of the listening room. Two different wave field representations have been introduced in Section 2.2: the decomposition into plane waves and circular harmonics. Either representation is a suitable choice for the wave domain as long as the components of the representation will result only in contributions of the same component after analysis. Consider now typical listening rooms with a rectangular shape a first approximation and their response to either plane waves or circular waves.

The response to a plane wave of a certain angle then consists in a mixture of plane waves with all kinds of different angles. This means that the desired effect of decoupling is not achievable with plane waves in more or less rectangular rooms.

On the other hand, the response to a circular wave depends on the wave length. Since high frequencies can be damped effectively by passive methods, active room compensation can be restricted to low frequencies. For the corresponding long wave lengths, scattering at the corners of the listening room does not play a major role. Therefore each circular harmonics component emitted by a well designed WFS system will lead to reflections which can be described mainly by the same component. Consequently the desired decoupling effect will be more exposed than for plane waves. This rough estimation suggests that circular harmonics seem to provide a more suitable choice for the wave domain representation than plane waves. This assumption holds also for reverberant rooms as long as the energy of other components than

the emitted one is considerably attenuated. The following section will specialize the generic concept of WDAF to the use of circular harmonics for the signal representation in the wave domain.
To this end the transformations $\mathscr{T}_1$, $\mathscr{T}_2$ and $\mathscr{T}_3$ introduced in Section 2.3 are expressed in terms of circular harmonics. Transformation $\mathscr{T}_1$ transforms the virtual source signal $Q(\omega)$, together with its spatial characteristics, into its representation in circular harmonics. The transformed signals are then pre-filtered by the room compensation filters $\check{\mathbf{C}}$. Transformation $\mathscr{T}_2$ computes suitable loudspeaker signals from the pre-filtered transformed signal components. The circular harmonics decomposition, as given by equation (1), can be used for this purpose. Block $\mathscr{T}_3$ transforms the microphone array signals $\mathbf{l}$ into their circular harmonics representation $\check{\mathbf{l}}$. A suitable transformation for circular microphone arrays was introduced in [16].

3. PERFORMANCE MEASURES

The WDAF concept relies on several properties of the transformations $\mathscr{T}_1$, $\mathscr{T}_2$ and $\mathscr{T}_3$ with respect to their ability to transform the MIMO adaptation problem into a sparse representation. The circular harmonics representation is based on the free-field eigensolutions of the wave equation. It therefore provides a perfect decoupling of the free-field transfer matrix but only an approximate decoupling of the listening room transfer matrix. Hence, it is sufficient to consider the properties of the room transfer matrix in its various representations in order to evaluate the performance of the circular harmonics decomposition as wave domain representation. The following performance measures are introduced for this purpose:

Singular Vectors It was stated in Section 2.4, that circular harmonics provide a reasonable basis for the WDAF approach. An indication for the suitability of this choice can be found by calculating the right singular vectors of the room transfer matrix $\underline{\mathbf{R}}(\omega)$ (for the pressure microphones) using the GSVD. The singular vectors will be sorted by their descending singular values, hence by their energy.

Energy compaction The ability of the different transformed signal and system representations to compact the room characteristics into less coefficients than using the microphone signals directly will be evaluated by two measures: (1) the energy of the elements of the room transfer matrix and (2) the energy compaction performance.
Four different representations of the room transfer matrix are considered: the representation in the pressure domain, by plane waves, by circular harmonics and the optimal decoupling given by the GSVD. The first measure is defined by calculating the energy of each spatial transmission path. For its representation in the pressure domain (pressure microphones) the energy of the elements of the room transfer matrix is defined as

$$E(m,n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |R_{m,n}(\omega)|^2 \, d\omega \,, \qquad (2)$$

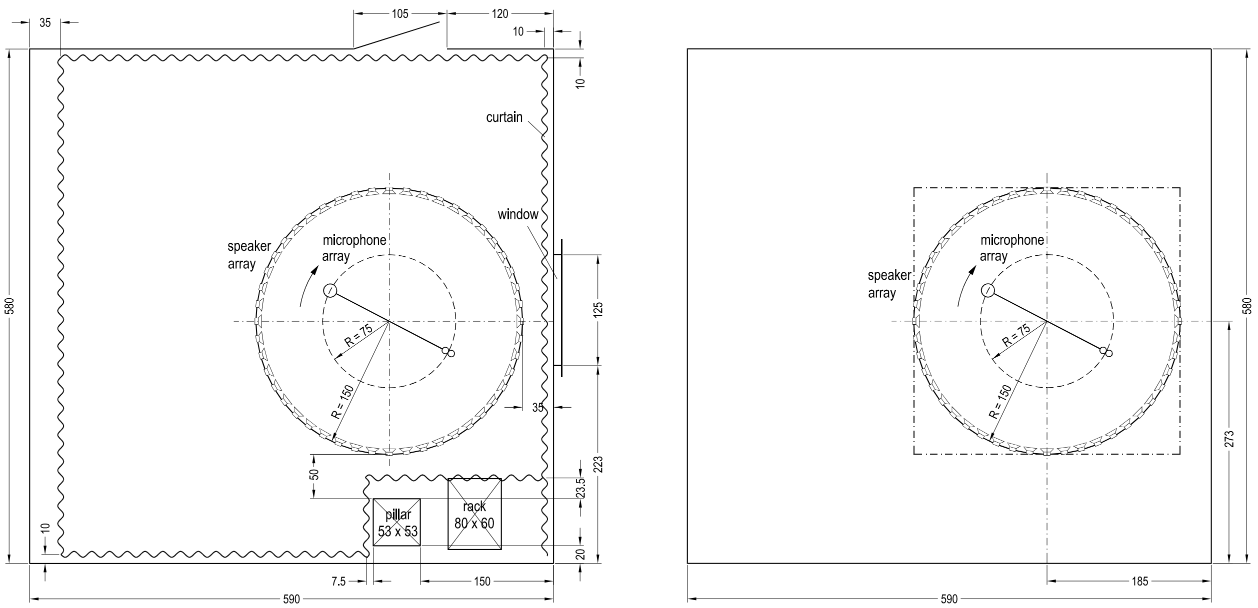
where $R_{m,n}(\omega)$ denotes the m,n-th element of the $M \times N$ room transfer matrix $\underline{\mathbf{R}}(\omega)$. Similar definitions apply to the room transfer matrix in its plane wave and circular harmonics representation yielding the energy representations $\bar{E}(\theta,\theta_0)$ and $\check{E}(\nu,\nu_0)$. Here the variables θ_0 and ν_0 denote the reproduced plane wave with incidence angle θ_0 or circular harmonic with order ν_0.
The energy compaction performance of the different transformations is evaluated by the energy compaction $\mathrm{EC}(i)$. For one particular room transfer matrix this measure is defined by calculating the ratio between the energy of the first i dominant elements and all elements. For this purpose the energies $E(m,n)$ are sorted in descending order yielding the sorted elements $E_{\mathrm{sort}}(\eta)$. Then the ratio between the energy of the first i sorted elements and all elements is calculated as follows

$$\mathrm{EC}(i) = \frac{\sum_{\eta=0}^{i} E_{\mathrm{sort}}(\eta)}{\sum_{\eta=0}^{MN-1} E_{\mathrm{sort}}(\eta)} \,, \qquad (3)$$

where $0 \leq \mathrm{EC}(i) \leq 1$. Equivalent definitions apply for the transformed representations. The more energy is captured by the first i elements the better the performance in terms of energy compaction.
The energy of the elements $E(m,n)$ illustrates the distribution of the energy in the different transformed domains, while $\mathrm{EC}(i)$ illustrates the ability of a transformation to compact the energy on few coefficients. The optimal wave domain transformation (e. g. GSVD-based) would compact the entire energy onto the main diagonal.

4. EVALUATION OF CIRCULAR HARMONICS DECOMPOSITION AS WAVE DOMAIN REPRESENTATION

The performance of the circular harmonics decomposition, as wave domain representation, will be evaluated



(a) Geometry of the multimedia laboratory

(b) Simplified geometry used for the simulation. The dashed line indicates the position of the rectangular loudspeaker array.

Fig. 2: Geometry of the measured/simulated listening room, loudspeaker and microphone array. The dimensions are given in centimeters [cm].

in the sequel by considering two different WFS systems: (1) a circular and (2) a rectangular one. The first setup consists of a 48 channel circular WFS system with a radius of $R_{\mathrm{LS}} = 1.50$ m. The system is realized with 48 two-way high-quality loudspeakers. The second setup consists of a rectangular WFS system. This system consists of 60 loudspeakers placed equidistantly at a distance of $\Delta x \approx 0.27$ m on a rectangular contour with the dimensions 3.50×3.50 m. The distance of the loudspeakers was chosen such to result in approximately the same spatial aliasing properties as the circular WFS system. The size and position of the loudspeaker arrays is depicted in Fig. 2.

The WFA system used for analyzing the reproduced wave field consists of 48 angular sampling positions with a radius of $R_{\mathrm{mic}} = 0.75$ m. The measurements were taken at the sampling positions in a sequential fashion using a stepper motor unit with a rod mounted onto it. Both a pressure and a pressure-gradient microphone were used for the measurements [16]. They were mounted at opposite ends of the rod (see Fig. 3) and the measurements were post-processed to cope for the opposite positions.

The listening room chosen for the measurements was the multimedia laboratory of “Multimedia Communications and Signal Processing” at the University of Erlangen-Nuremberg [17]. The laboratory has the dimensions $5.90 \times 5.80 \times 3.10$ m $(\mathrm{w} \times \mathrm{l} \times \mathrm{h})$. The room is equipped with a carpet, a damped ceiling and sound absorbing curtains at all sides. The curtains have been removed fully for the measurements. As stated in [13] elevated reflections can neither be compensated nor analyzed properly by two-dimensional WFS and WFA systems. The floor was therefore damped additionally by damping material (see Fig. 3) placed below the WFS system. Figure 2(a) illustrates the geometry of the multimedia laboratory. The geometry of the simulated acoustical environment is a simplified version of the multimedia laboratory due to complexity considerations. The simulated geometry is depicted by Fig. 2(b).

For the evaluation of the transformation the impulse responses from all loudspeakers to all (pressure/pressure-gradient) microphones were measured/simulated, in order to derive the required room transfer matrix. Figure 3 illustrates the measurement setup. The plane wave reflection factor R_{pw} of the walls in the simulation was varied from $R_{\mathrm{pw}} = 0 \ldots 1$ in $\Delta R_{\mathrm{pw}} = 0.05$ steps. How-

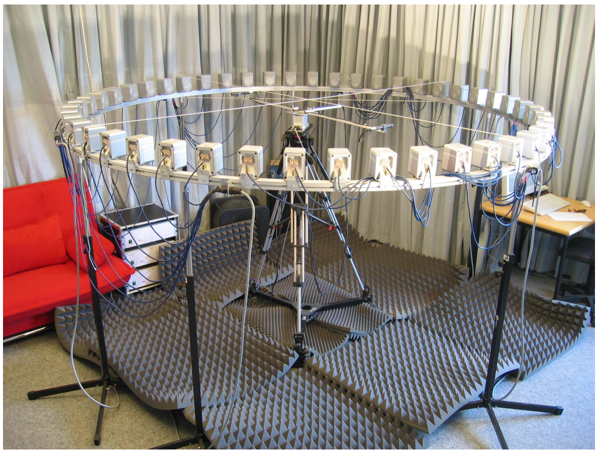


Fig. 3: Setup used for the measurement of the listening room transfer matrix. The gray curtains were removed for the measurements discussed in Section 4.2.

ever, the results will only be shown for two cases: a free-field scenario with $R_{\text{pw}} = 0$ and a reverberant scenario with $R_{\text{pw}} = 0.8$. The latter one approximates the acoustic characteristics of the multimedia laboratory (with all curtains opened).

Both reproduction systems were numerically simulated and the circular WFS system additionally measured in-situ. The main benefits of using simulated versus real acoustical environments are that the parameters of the simulated environment can be changed easily in order to simulate different scenarios and that practical aspects as noise, transducer mismatch and misplacement can be excluded for a first proof of the WDAF concept.

All signals were captured at a sampling rate of 48 kHz and then low-pass filtered to a frequency of $f_{\text{LP}} = 650$ Hz in order to keep the spatial aliasing artifacts reasonably small [14].

4.1. Simulated Acoustic Environment

The following section will evaluate the performance of the circular harmonics decomposition as wave domain transformation on the basis of simulated acoustical environments for the circular/rectangular WFS system.

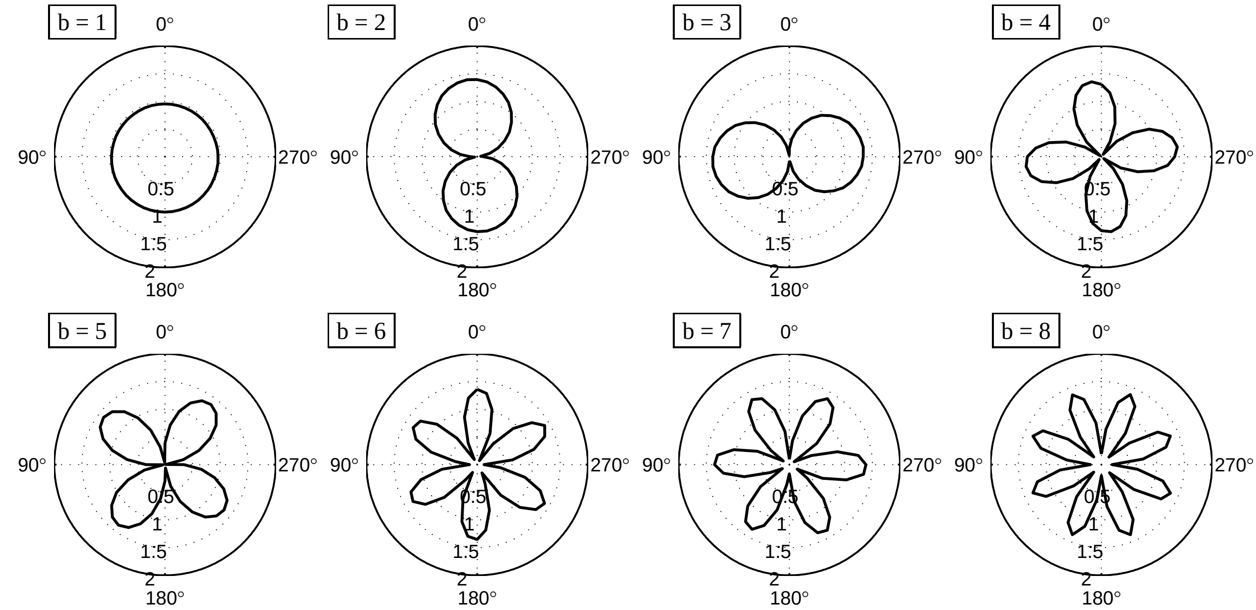
For an accurate simulation of the acoustic environment a numerical simulation of the wave equation based on the functional transformation method (FTM) is used [18]. The particular implementation utilized (`wave2d`) simulates the wave propagation in two-dimensions [19]. This method requires no spatial discretization and thus microphones and speakers can be placed at their exact spatial positions. This is not possible when using methods which perform a spatial discretization, like e. g. the finite element method (FEM).

4.1.1. Circular WFS System

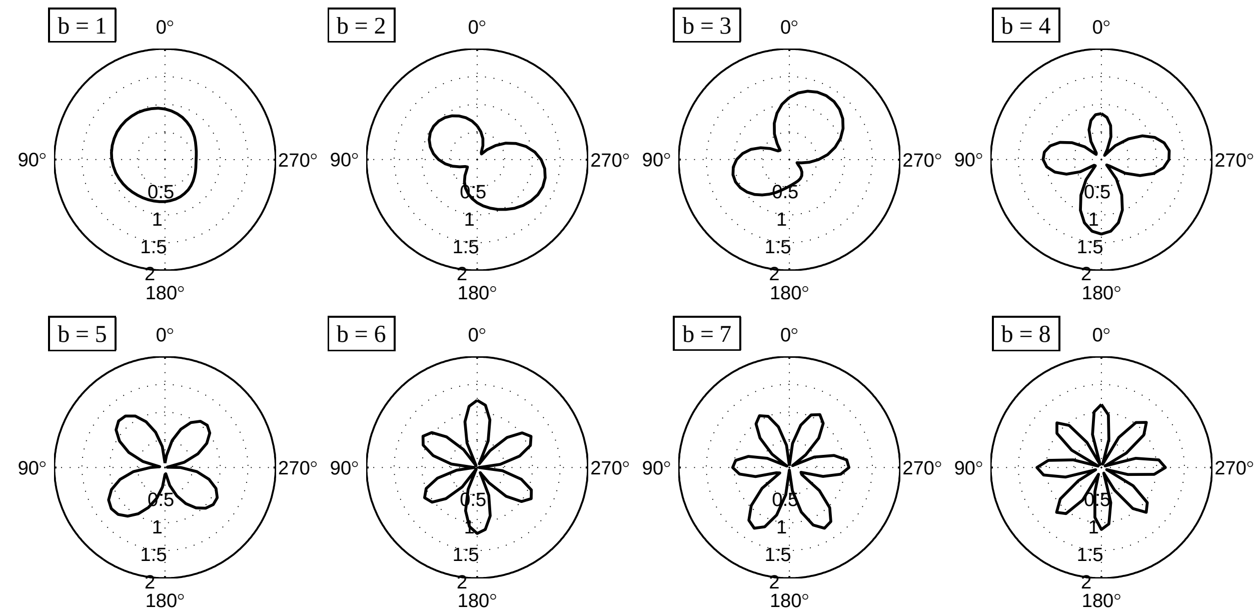
The performance of the circular harmonics decomposition is evaluated in the sequel for the circular WFS system by using the performance measures introduced in Section 3.

Singular Vectors Figure 4(a) shows the absolute value of the first eight right singular vectors for the free-field case ($R_{\text{pw}} = 0$) and a frequency of $f = 80$ Hz. The results are similar to the angular basis functions of the circular harmonics [15]. This conclusion is evident since free-field propagation has been simulated and circular harmonics match optimally to the geometry of the circular WFA system in this case. In order to prove the suitability of the circular harmonics decomposition for a typical listening room, the singular vectors were also computed for the reverberant case ($R_{\text{pw}} = 0.8$). These singular vectors are shown in Fig. 4(b). It can be seen that they are deformed to some extend compared to the circular harmonics, however their coarse structure still matches the circular harmonics quite well. The presented results indicate that circular harmonics provide a suitable transformation for typical listening rooms.

Energy Compaction Figure 5 shows the energy distribution of the room transfer matrix for the pressure microphones and its circular harmonics representation for the two different simulated scenarios. Figure 5(a) shows $E(m,n)$ for the pressure microphones in the free-field case ($R_{\text{pw}} = 0$). The direct path from the loudspeakers to the microphones can be seen clearly. Figure 5(b) shows $\breve{E}(\nu, \nu_0)$ for the free-field case. Almost all energy is compacted onto the main diagonal elements by the circular harmonics representation of the room transfer matrix. Please note the different scales used for the results shown. For the free-field case all energy in the circular harmonics domain should be compacted onto the main diagonal. However, some off diagonal elements with very low energy are also present in Fig. 5(b) due to aliasing and truncation artifacts present in the analyzed wave field and numerical errors in the free-field simulation. Figure 5(c) and 5(d) show the energy distributions for the reverberant case. The reflections of the loudspeaker wave fields at the walls of the listening room can be seen

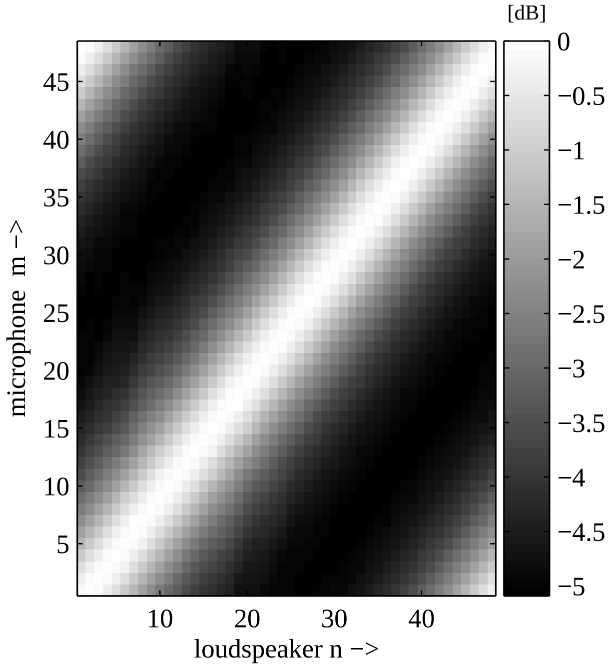


(a) reflection factor $R_{\mathrm{pw}} = 0$

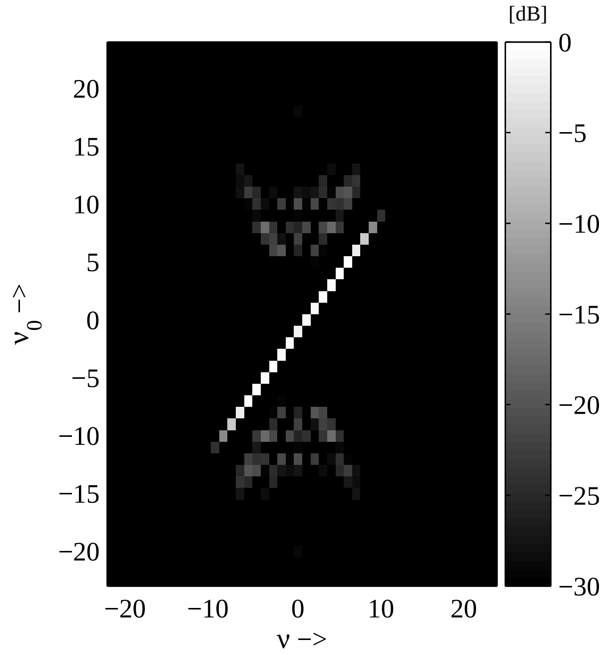


(b) reflection factor $R_{\mathrm{pw}} = 0.8$

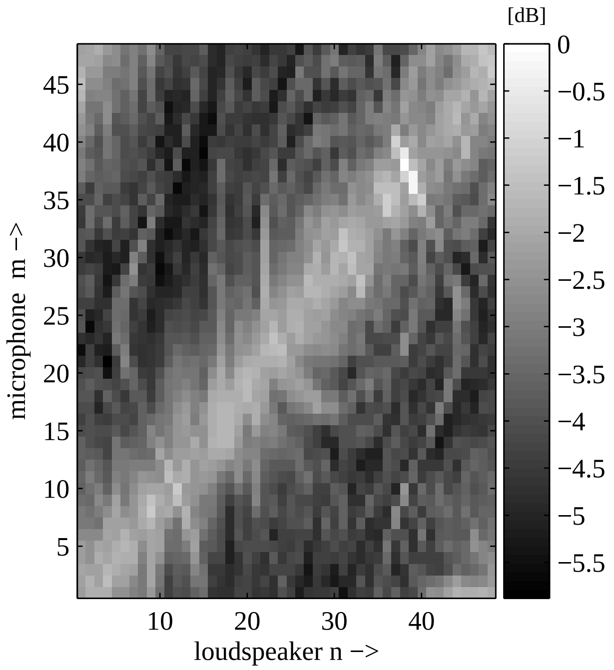
Fig. 4: Absolute value of the first eight right singular vectors ($f = 80$ Hz) for the simulated circular WFS/WFA system sorted by their descending singular values (top left to bottom right). The singular vectors for two different plane wave reflection factors R_{pw} off the walls of the simulated room are shown.



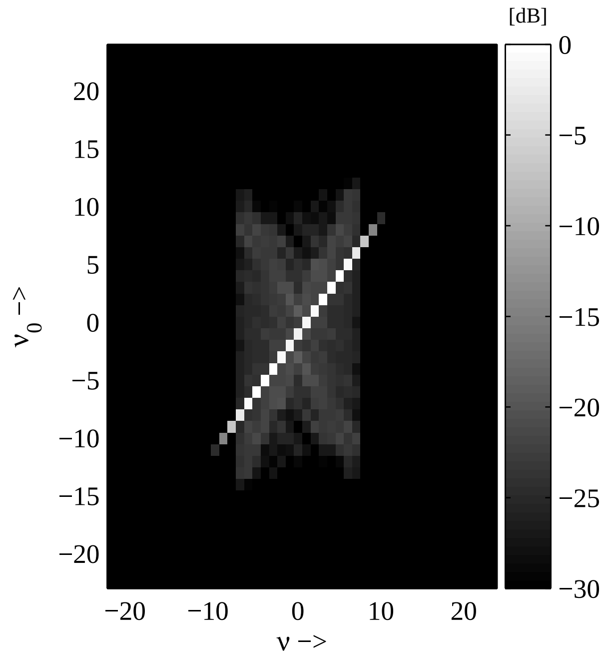
(a) pressure domain $R_{\text{pw}} = 0$



(b) circular harmonics domain $R_{\text{pw}} = 0$



(c) pressure domain $R_{\text{pw}} = 0.8$



(d) circular harmonics domain $R_{\text{pw}} = 0.8$

Fig. 5: Energy of the room transfer matrix of the signals captured by the pressure microphones $E(m,n)$ and in the circular harmonics domain $\breve{E}(\nu,\nu_0)$ for the simulated circular WFS/WFA system. The two rows show the results for different plane wave reflection factors R_{pw} at the walls of the simulated room.

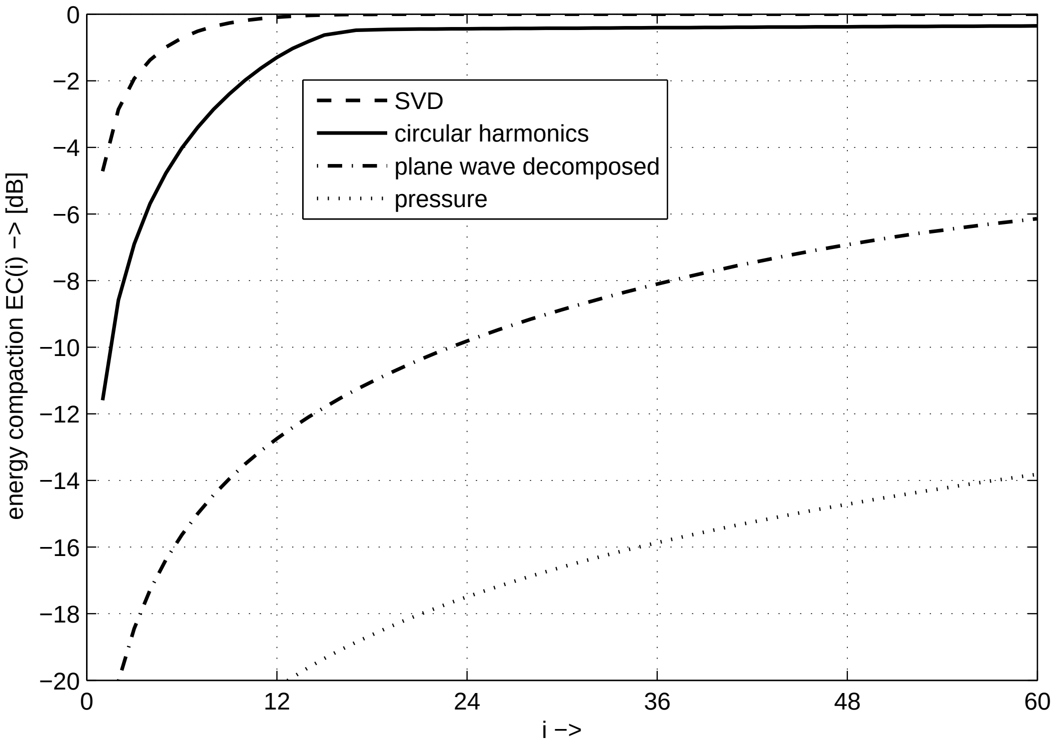


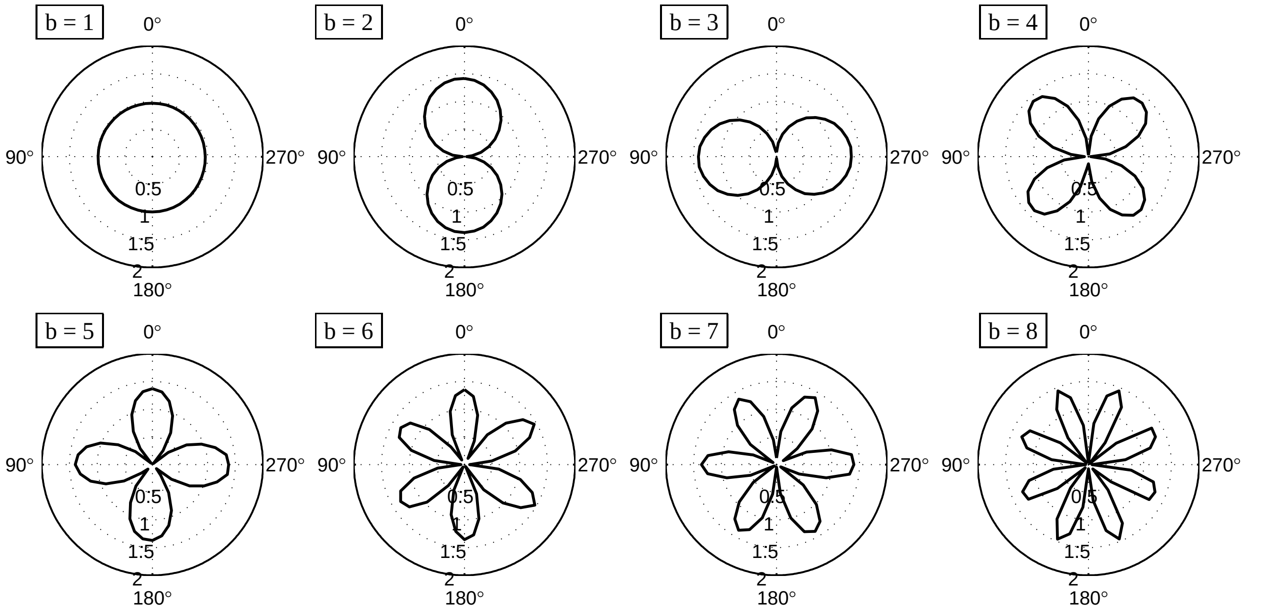
Fig. 6: Energy compaction performance $\mathrm{EC}(i)$ for the different representations of the room transfer matrix for the simulated circular WFS/WFA system. Shown are the results for $R_{\mathrm{pw}} = 0.8$

clearly in Fig. 5(c). Figure 5(d) shows the energy of the room transfer matrix represented in circular harmonics. As desired, the main diagonal elements represent a major portion of the energy. The off-diagonal elements are a result of the reverberant enclosure.
Not only the performance of the circular harmonics in terms of energy compaction towards the main diagonal is of interest, but also its performance to represent the listening room by as few coefficients as possible. This can be measured by the energy compaction of the different representations. Results for the reverberant case are shown in Fig. 6: energy compaction according to Eq. (3) for the room transfer matrix in the pressure domain $\underline{\mathrm{EC}}(i)$, in the plane wave decomposed domain $\overline{\mathrm{EC}}(i)$ and in the circular harmonics domain $\check{\mathrm{EC}}(i)$ for the simulated circular WFS/WFA system. The energy compaction of the optimal (GSVD-based) transformation is shown for reference. It can be seen clearly that the circular harmonics representation of the room transfer matrix compacts the energy much better than its pressure and plane wave representation. It can be seen also that the GSVD provides the optimal transformation in this sense. The presented results indicate that the circular harmonics decomposition is not the optimal transformation for the reverberant case. However, the results also prove that this transformation provides a quite reasonable approximation.

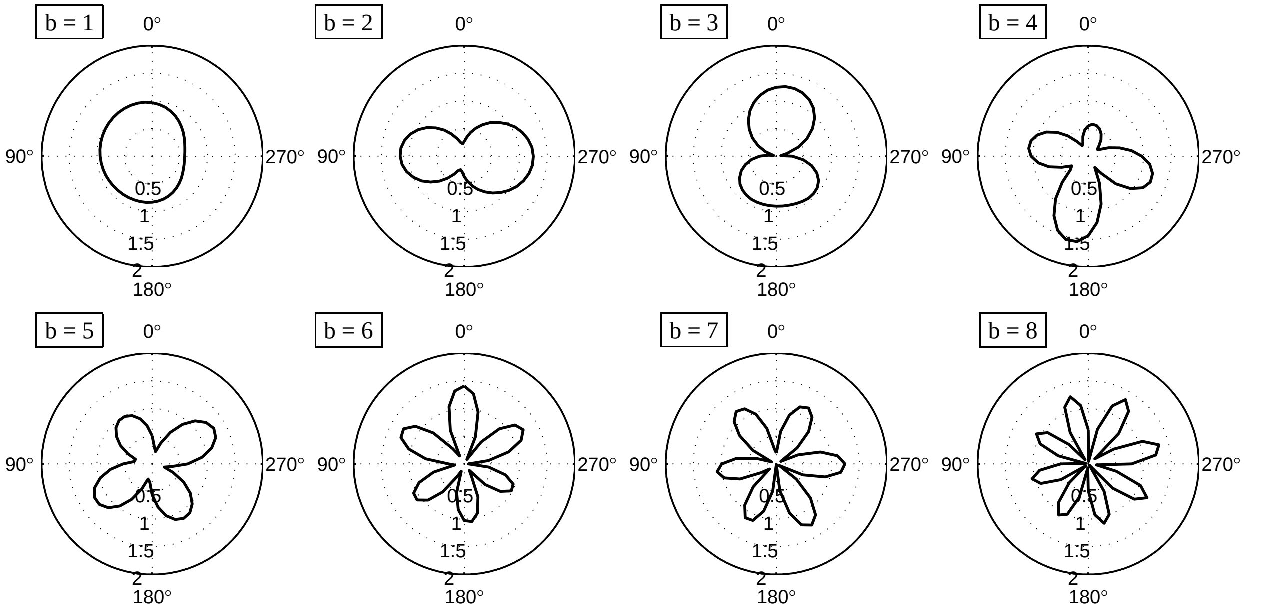
4.1.2. Rectangular WFS System

The performance of the circular harmonics decomposition is evaluated additionally for a rectangular WFS system in order to illustrate its invariance with regard to the geometry of the reproduction system.

Singular Vectors Figure 7(a) shows the absolute value of the first eight right singular vectors for the free-field case ($R_{\mathrm{pw}} = 0$) and a frequency of $f = 80$ Hz. The singular vectors for the reverberant case ($R_{\mathrm{pw}} = 0.8$) are shown in Fig. 7(b). It can be seen that they are deformed to some extend in comparison with the circular harmonics,



(a) reflection factor $R_{\mathrm{pw}} = 0.0$



(b) reflection factor $R_{\mathrm{pw}} = 0.8$

Fig. 7: Absolute value of the first eight right singular vectors ($f = 80$ Hz) for the simulated rectangular WFS/circular WFA system sorted by descending singular values (top left to bottom right). The singular vectors for two different plane wave reflection factors R_{pw} at the walls of the simulated room are shown.

however their coarse structure still matches the circular harmonics. As expected, the presented results indicate that circular harmonics provide a suitable transformation for the rectangular WFS system also.

Energy Compaction Figure 8 shows the energy distribution and compaction of the room transfer matrix in its different representations for the reverberant case. Please note the different scales used for the results shown. Figure 8(a) shows $E(m,n)$ for the pressure microphones and Fig. 8(b) shows the energy $\breve{E}(\nu,\nu_0)$ of the room transfer matrix represented in circular harmonics. As desired, the main diagonal elements in the circular harmonics domain represent a major portion of the energy. The energy compaction of the different representations for the reverberant case is shown in Fig. 8(c). It can be seen clearly that the circular harmonics representation of the room transfer matrix compacts the energy much better than its pressure or plane wave representation.
As for the circular WFS system, these results prove that the circular harmonics decomposition of the room transfer matrix provides the desired properties. This result is evident since an optimally designed WFS system should have no influence on the transformation used for WDAF.

4.2. Measured Acoustic Environment

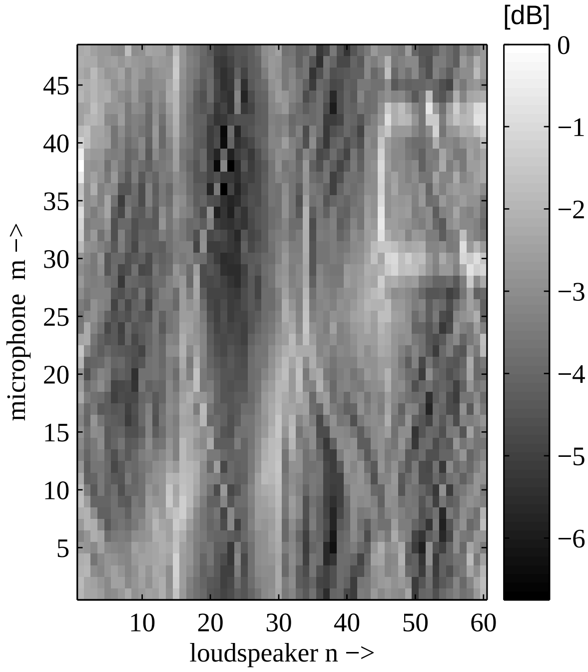
Up to now, two-dimensional simulations of acoustic environments were used to evaluate the performance of the circular harmonics decomposition as wave domain representation. To get insight into its performance in real-world applications, measurements have been taken from the laboratory WFS/WFA system shown in Fig. 3. The following section will discuss the derived results.

Singular Vectors Figure 9 shows the absolute value of the first eight right singular vectors. Shown are the singular vectors for a frequency of $f = 80$ Hz. The singular vectors are quite deformed compared to the simulation results shown in Fig. 4(b). However, their coarse structure is still equivalent to the circular harmonics. Reasons for the deformations are measurement and ambient noise, variations in the directivity and frequency response of the microphones used compared to ideal omnidirectional and figure-of-eight microphones and elevated reflections.

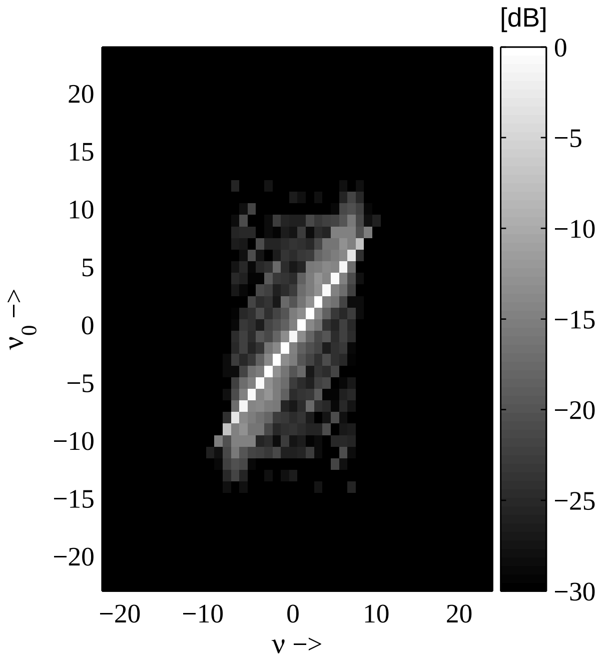
Energy Compaction Figure 10 shows the energy distribution and compaction of the room transfer matrix in its different representations for the measured setup. Please note the different scales used. Figure 10(a) shows the energy distribution $E(m,n)$ of the room transfer matrix in the pressure domain. Figure 10(b) shows its energy distribution $\breve{E}(\nu,\nu_0)$ in the circular harmonics domain. Here the main diagonal elements represent a major portion of the energy. The off-diagonal elements are a result of the reverberant listening room and the measurement errors mentioned before. The structure of the energy representation in the circular harmonics domain is quite similar to the simulation results shown in Fig. 5(d). This indicates that the simulations can be used quite well to predict the real-world performance.
The energy compaction of the different representations for the measured room is shown in Fig. 10(c). The energy compaction performance of the circular harmonics for the measured setup is only slightly lower than for the simulated setup shown in Fig. 6.
Summarizing, the results obtained from the measurement of the circular WFS/WFA setup prove that the circular harmonics decomposition of the room transfer matrix provides the desired properties for the WDAF concept. Hence, the circular harmonics decomposition is a reasonable transformation for WDAF based active room compensation in real-world applications.

5. CONCLUSIONS

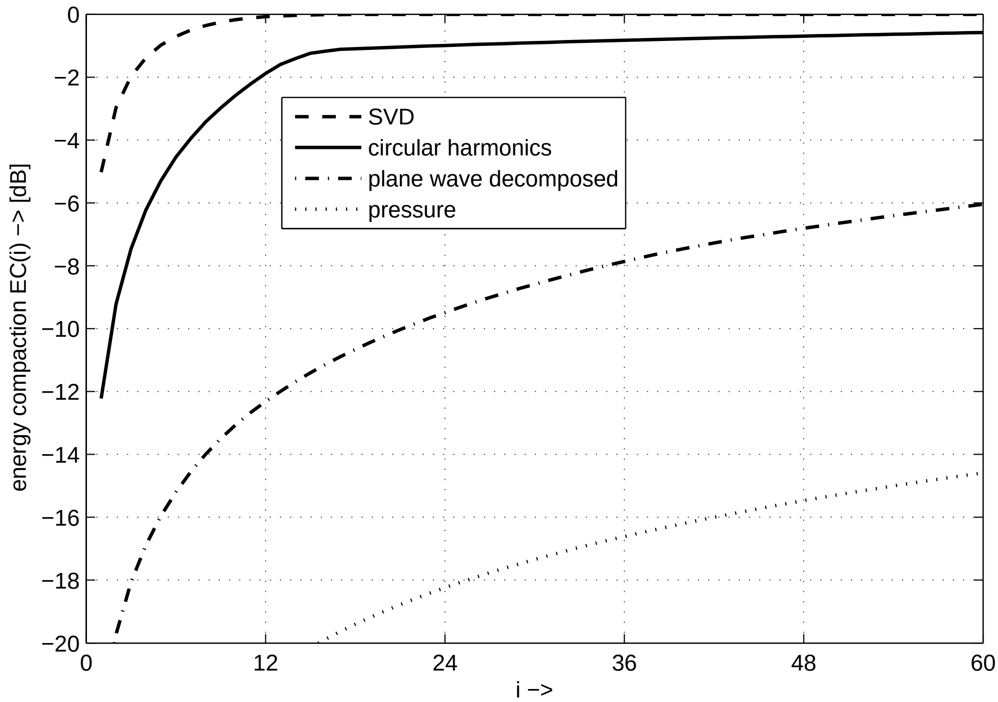
In the context of massive multichannel systems, the concept of WDAF provides a versatile framework for adaptive signal processing. It is based upon a combination of principles from transform coding, wave propagation and adaptive signal processing to improve the adaptation process in terms of complexity and numerical stability. However, a suitable wave domain transformation has to be found for a particular adaptation problem. It has been shown [5] that the optimal transformation is given by a GSVD of the free-field and the actual room transfer matrix. Since the room transfer characteristics are in general not known or may change over time, wave domain transformations which are independent from these transfer characteristics are desirable. In general, such a data-independent transformation will not provide an optimal decoupling of the MIMO adaptive system. However, if the transformation is chosen such to represent the multichannel adaptation problem by its main diagonal and some additional off-diagonal paths then the adaptation process will still be improved highly by the WDAF concept.
It has been proposed to use the circular harmonics



(a) energy distribution in the pressure domain



(b) energy distribution in the circular harmonics domain



(c) energy compaction performance

Fig. 8: Energy distribution and compaction of the room transfer matrix in its different representations for the simulated rectangular WFS/circular WFA system. Shown are the results for $R_{\text{pw}} = 0.8$.

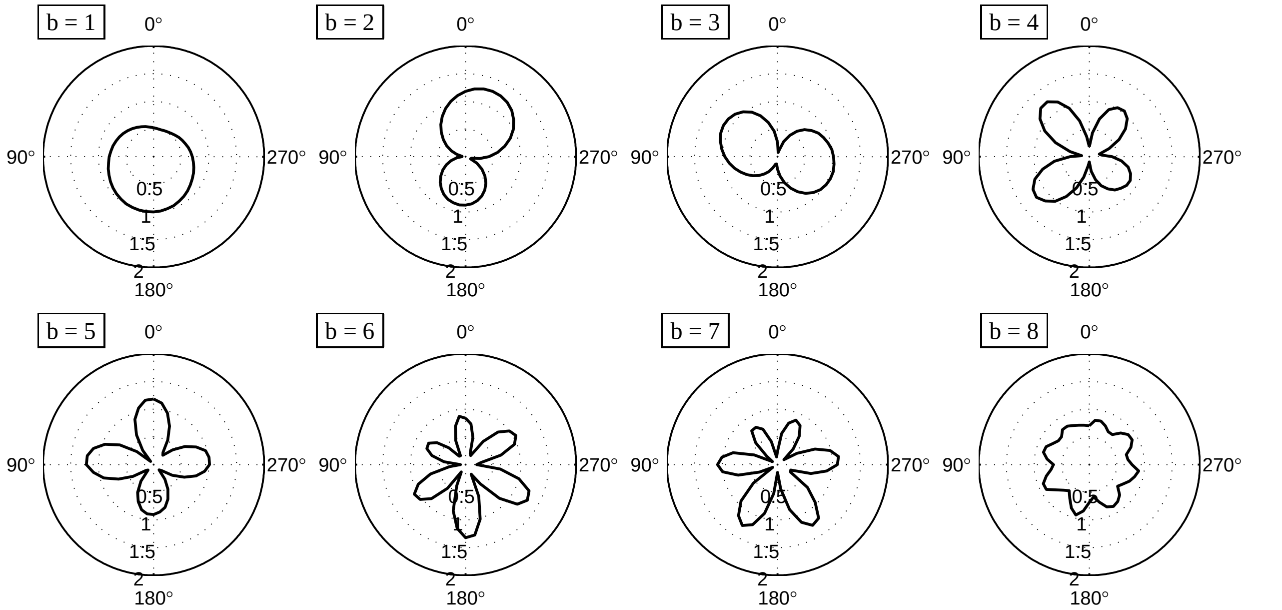


Fig. 9: Absolute value of the first eight right singular vectors ($f = 80$ Hz) of the measured circular WFS/WFA system sorted by descending singular values (top left to bottom right).

decomposition as wave domain transformation for the adaptation of acoustic massive multichannel systems placed in typical rooms [6, 7, 8]. This decomposition is based upon the free-field eigensolutions of the wave equation. It therefore provides an optimal decoupling of the free-field transfer matrix $\underline{\mathbf{F}}(\omega)$ but in general not for the listening room transfer matrix $\underline{\mathbf{R}}(\omega)$. The major benefit of using the circular harmonics decomposition instead of a GSVD-based transformation for the WDAF concept is its data independence.
This paper presented an analysis of the circular harmonics decomposition in the the context of WDAF applied to acoustic massive multichannel systems. This analysis was performed by simulating and measuring combined WFS/WFA systems placed in a typical listening room and analyzing the resulting properties of the room transfer matrix in the transformed domain. The results proved that this special choice for the wave domain transformation provides the desired property of energy compaction. Additional simulations performed within the course of our research with different virtual listening rooms and reproduction geometries revealed a similar performance as presented here. This indicates that the circular harmonics decomposition also provides a suitable transformation for other reproduction scenarios.

6. REFERENCES

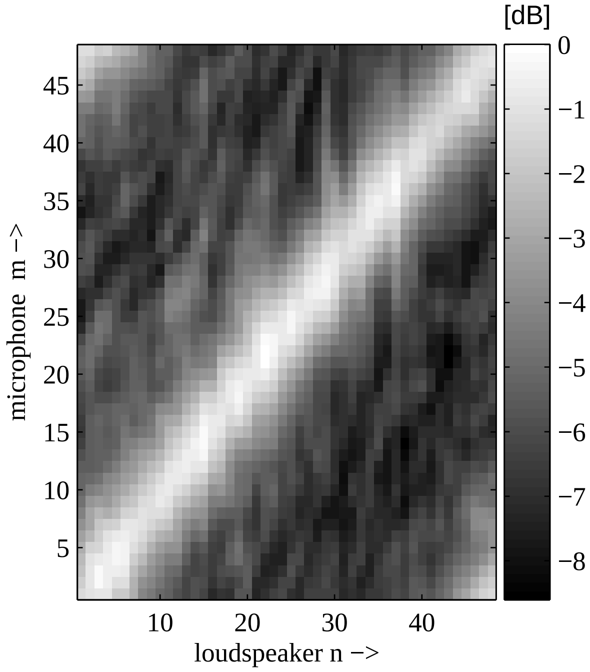
[1] A.J. Berkhout. A holographic approach to acoustic control. *Journal of the Audio Engineering Society*, 36:977–995, December 1988.

[2] E. Hulsebos. *Auralization using Wave Field Synthesis*. PhD thesis, Delft University of Technology, 2004.

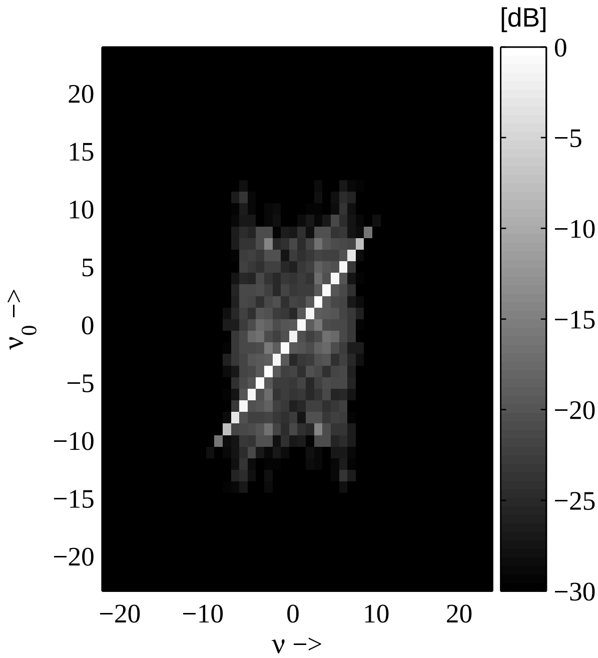
[3] J. Daniel. *Représentation de champs acoustiques, application à la transmission et à la reproduction de scènes sonores complexes dans un contexte multimédia*. PhD thesis, Université Paris 6, 2000.

[4] M.A. Gerzon. Ambisonics in multichannel broadcasting and video. *Journal of the Acoustic Society of America*, 33(11):859–871, Nov. 1985.

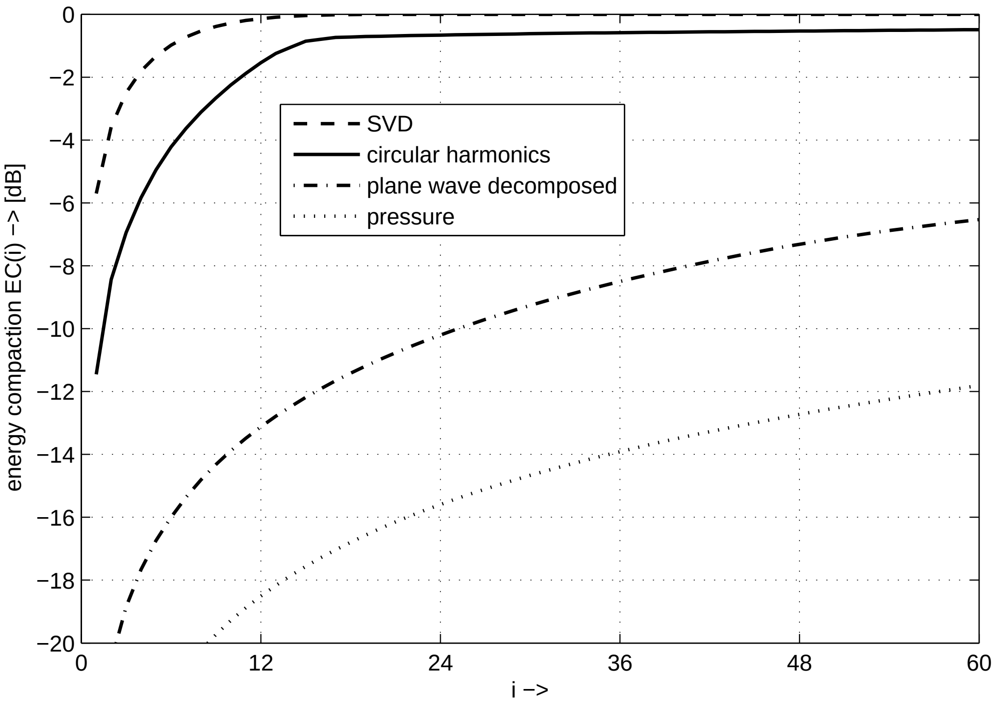
[5] S. Spors, H. Buchner, and R. Rabenstein. Eigenspace adaptive filtering for efficient pre-equalization of acoustic MIMO systems. In *Eu-*



(a) energy distribution in the pressure domain



(b) energy distribution in the circular harmonics domain



(c) energy compaction performance

Fig. 10: Energy distribution and compaction of the room transfer matrix in its different representations for the measured circular WFS/WFA setup.

ropean Signal Processing Conference (EUSIPCO), 2006. To be published.

[6] S. Spors, H. Buchner, and R. Rabenstein. Efficient active listening room compensation for Wave Field Synthesis. In *116th AES Convention*, Berlin, Germany, 2004. Audio Engineering Society (AES).

[7] H. Buchner, S. Spors, and W. Kellermann. Full-duplex systems for sound field recording and auralization based on Wave Field Synthesis. In *116th AES Convention*, Berlin, Germany, 2004. Audio Engineering Society (AES).

[8] W. Herbordt, S. Nakamura, S. Spors, H. Buchner, and W. Kellermann. Wave field cancellation using wave-domain adaptive filtering. In *Hand-Free Speech Communication and Microphone Arrays*, New Jersey, USA, 2005.

[9] A.J. Berkhout, D. de Vries, and P. Vogel. Acoustic control by wave field synthesis. *Journal of the Acoustic Society of America*, 93(5):2764–2778, May 1993.

[10] P.M. Morse and H. Feshbach. *Methods of theoretical physics. Part I.* McGraw-Hill, New York, 1953.

[11] D. de Vries, E.W. Start, and V.G. Valstar. The Wave Field Synthesis concept applied to sound reinforcement: Restrictions and solutions. In *96th AES Convention*, Amsterdam, Netherlands, February 1994. Audio Engineering Society (AES).

[12] J.-J. Sonke, D. de Vries, and J. Labeeuw. Variable acoustics by wave field synthesis: A closer look at amplitude effects. In *104th AES Convention*, Amsterdam, Netherlands, May 1998. Audio Engineering Society (AES).

[13] S. Spors, M. Renk, and R. Rabenstein. Limiting effects of active room compensation using wave field synthesis. In *118th AES Convention*, Barcelona, Spain, May 2005. Audio Engineering Society (AES).

[14] S. Spors and R. Rabenstein. Spatial aliasing artifacts produced by linear and circular loudspeaker arrays used for wave field synthesis. In *120th AES Convention*, Paris, France, 2006. Audio Engineering Society (AES).

[15] E.G. Williams. *Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography*. Academic Press, 1999.

[16] E. Hulsebos, D. de Vries, and E. Bourdillat. Improved microphone array configurations for auralization of sound fields by Wave Field Synthesis. In *110th AES Convention*, Amsterdam, Netherlands, May 2001. Audio Engineering Society (AES).

[17] Multimedia Communications and Signal Processing at the University of Erlangen-Nuremberg. `http://www.lnt.de/LMS`.

[18] S. Petrausch and R. Rabenstein. Highly efficient simulation and visualization of acoustic wave fields with the functional transformation method. In *Simulation and Visualization*, pages 279–290, Magdeburg, March 2005. Otto von Guericke Universität.

[19] Wave2D: Wave field simulation in 2D. `http://www.lnt.de/~stepe/demo/Wave2D`.