

AN ANALYTICAL APPROACH TO 2.5D SOUND FIELD REPRODUCTION EMPLOYING LINEAR DISTRIBUTIONS OF NON-OMNIDIRECTIONAL LOUDSPEAKERS

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ABSTRACT

We present an analytical approach targeting the physical reproduction of sound fields by means of linear distributions loudspeakers. Unlike with conventional analytical approaches like wave field synthesis, the employed loudspeakers are not required to be omnidirectional. The approach does not compensate for loudspeaker properties which deviate from certain assumptions but it rather allows for the explicit consideration of these properties within some limits which are outlined.

Index Terms— spatial Fourier transform, wave field synthesis, spectral division method

1. INTRODUCTION

Traditionally, massive-multichannel sound field reproduction methods like wave field synthesis or higher order Ambisonics assume that the involved secondary sources (i.e. loudspeakers) are omnidirectional. For lower frequencies, this assumption is indeed approximately fulfilled when conventional loudspeakers with closed cabinets are considered. However, for higher frequencies above a few thousand Hertz complex radiation patterns evolve.

A number of approaches based on the theory of multiple-input-multiple-output (MIMO) systems have been proposed in order to compensate for loudspeaker radiation characteristics and influence of the reproduction room (loudspeaker radiation characteristics are part of the latter). Approaches suitable for linear loudspeaker arrays are e.g. [1, 2, 3, 4].

Room compensation requires realtime analysis of the reproduced sound field and adaptive algorithms due to the time-variance of room acoustics (e.g. temperature variations). Compensation of the loudspeaker radiation characteristics, such as directivity and frequency response, is less complex since it can be assumed that these characteristics are time-invariant. No adaptation and therefore no real-time analysis is required. However, in order that the radiation characteristics can be compensated for while neglecting the reproduction room, the radiation characteristics of the entire secondary source setup have to be measured under anechoic conditions.

When certain physical constraints are accepted, a significant reduction of complexity can be achieved. Besides time-invariance, the fundamental physical constraints introduced in the presented approach are:

- (1) The secondary source arrangement is linear.
- (2) The spatio-temporal transfer function of the secondary sources is shift invariant. In other words, all individual secondary sources have to have equal radiation characteristics and have to be equally orientated.

Requirement (1) can obviously be fulfilled. Preliminary measurements undertaken at Deutsche Telekom Laboratories have shown that typical commercially available loudspeakers with closed cabinets indeed exhibit similar to equal spatio-temporal transfer functions in anechoic conditions. This suggests that requirement (2) can also be fulfilled when the acoustical properties of the reproduction room are ignored.

The approach treated in this paper is termed *spectral division method* and has been presented for linear and planar secondary source distributions by the authors in [5]. A formulation of the spectral division method for spherical and circular secondary source distributions has been presented by the authors in [6, 7].

Unlike MIMO approaches which are discrete by nature, the spectral division method is continuous and enables the application of powerful analytical tools for the analysis of its general properties and limitations.

In this contribution, we restrict the considerations to linear distributions of secondary sources and focus on the incorporation of secondary sources with given radiation characteristics. The properties of the reproduced sound field are investigated analytically and numerical simulations are presented which illustrate the results.

Nomenclature The following notational conventions are used: For scalar variables, upper case denotes the temporal frequency domain. The spatial frequency domain (wavenumber domain) is indicated by a tilde over the respective symbol. The dependent variables of a given quantity in the spatial frequency domain indicate with respect to which dimension the spatial frequency domain is considered. E.g. $\tilde{P}(k_x, y, z, \omega)$ means that $P(\mathbf{x}, \omega)$ is considered in the wavenumber domain only with respect to k_x .

Vectors are denoted by lower case boldface. The three-dimensional position vector is given by $\mathbf{x} = [x \ y \ z]^T$ and by (r, α, β) in spherical coordinates. Refer also to the coordinate systems depicted in Fig. 1. The acoustic wavenumber is denoted by k . It is related to the temporal frequency by $k^2 = (\frac{\omega}{c})^2$ with $\omega = 2\pi f$ being the radial frequency and c the speed of sound.

Monochromatic plane waves are denoted by $e^{-j\mathbf{k}_{pw}^T \mathbf{x}}$, with $\mathbf{k}_{pw}^T = [k_{pw,x} \ k_{pw,y} \ k_{pw,z}] = k_{pw} \cdot [\cos \theta_{pw} \sin \phi_{pw} \ \sin \theta_{pw} \sin \phi_{pw} \ \cos \phi_{pw}]$ and (θ_{pw}, ϕ_{pw}) being the propagation direction of the plane wave. j is the imaginary unit ($j = \sqrt{-1}$).

We refer to secondary sources rather than to loudspeakers since we assume their distributions to be continuous throughout this paper.

2. DERIVATION OF THE SECONDARY SOURCE DRIVING FUNCTION

In order to analyze the properties of the sound field reproduced by planar and linear secondary source distributions, we have to find the

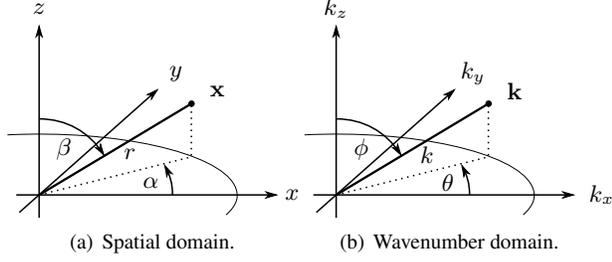


Fig. 1. The coordinate systems used in this paper.

appropriate secondary source driving signals. The procedure is outlined in the following.

We will exemplarily derive the explicit driving signals to reproduce a virtual plane wave of given propagation direction and frequency. The obtained results can be straightforwardly extended to the reproduction of complex sound fields via the *angular spectrum representation* [8]. The latter represents the decomposition of wave fields into a continuum of plane waves in a source-free region. The appropriate combination of the driving signals for plane waves as indicated by the angular spectrum representation yields the driving signals for a given complex sound field to be reproduced.

For convenience, the secondary source array is assumed to be along the x -axis (thus $\mathbf{x}_0 = [x_0 \ 0 \ 0]^T$, refer to Fig. 2). For this setup the

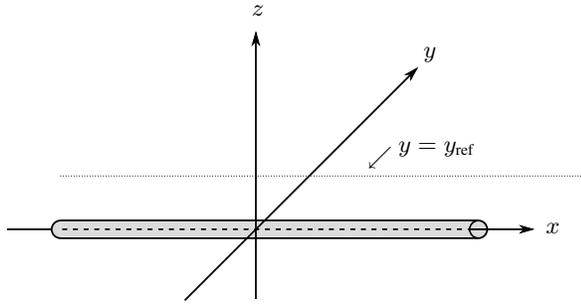


Fig. 2. Illustration of the geometrical setup. The secondary source distribution is situated along the x -axis. It is indicated by the grey shading and has infinite extend. The target half-plane is the half-plane bounded by the secondary source distribution and contains the positive y -axis. Thin dotted lines indicate the reference line (see text).

reproduction equation is given by [9]

$$P(\mathbf{x}, \omega) = \int_{-\infty}^{\infty} D(\mathbf{x}_0, \omega) \cdot G(\mathbf{x} - \mathbf{x}_0, \omega) dx_0. \quad (1)$$

Note that we assume $G(\cdot)$ to be shift invariant (we write $G(\mathbf{x} - \mathbf{x}_0, \omega)$ instead of $G(\mathbf{x}|\mathbf{x}_0, \omega)$) [8]. This requires that all secondary sources have to have equal spatio-temporal characteristics and orientation.

Equation (1) can be viewed as a convolution integral. This fact is revealed when (1) is rewritten as

$$P(\mathbf{x}, \omega) = \int_{-\infty}^{\infty} D(x_0, \omega) G\left([x \ y \ z]^T - [x_0 \ 0 \ 0]^T, \omega\right) dx_0 = (2)$$

$$= \int_{-\infty}^{\infty} D(x_0, \omega) G(x - x_0, y, z, \omega) dx_0 = \quad (3)$$

$$= D(x, \omega) *_x G(\mathbf{x}, \omega), \quad (4)$$

whereby $G(\mathbf{x}, \omega)$ represents the spatio-temporal transfer function of the secondary source located in the origin of the coordinate system. The convolution is performed along the x -axis and the convolution theorem

$$\tilde{P}(k_x, y, z, \omega) = \tilde{D}(k_x, \omega) \cdot \tilde{G}(k_x, y, z, \omega) \quad (5)$$

holds [10]. The secondary source driving function $\tilde{D}(k_x, \omega)$ in wavenumber domain is thus given by

$$\tilde{D}(k_x, \omega) = \frac{\tilde{P}(k_x, y, z, \omega)}{\tilde{G}(k_x, y, z, \omega)}. \quad (6)$$

In the above derivation, we intentionally assumed $D(x, \omega)$ to be exclusively dependent on x because x is the only degree of freedom in the position of the secondary sources. However, generally $D(x, \omega)$ will be dependent on the position of the receiver. This is mathematically reflected by the fact that y and z do not cancel out in (6) [5].

It is not surprising that we are not able to reproduce arbitrary sound fields over an extended area since we are dealing with a secondary source distribution which is neither infinite in two dimensions nor does it enclose the target volume [8].

In the present case, the secondary source setup will only be capable of creating wave fronts that propagate away from it. We will treat this circumstance in an intuitive way in the following. Refer to [5] for a rigorous derivation.

The propagation direction of the reproduced sound field can generally only be correct inside one half-plane bounded by the secondary source distribution. We term this half-plane *target half-plane*. The reproduced sound field anywhere else in space is a byproduct whose properties are determined by the secondary source driving function $D(x, \omega)$ and the radiation characteristics of the secondary sources in the respective direction. For convenience, we aim at reproducing a given desired sound field inside that half of the horizontal plane which contains the positive y -axis. We therefore set $z = 0$.

However, above considerations do not affect the dependence of the driving function on y . In other words, even inside the target half-plane the reproduced sound field will generally only be correct on a line parallel to the x -axis at distance $y = y_{\text{ref}}$ [5]. At locations off this reference line, the reproduced sound field generally deviates from the desired sound field in terms of amplitude, propagation direction, and near-field components [5].

The present situation, i.e. the employment of secondary sources with a three-dimensional spatio-temporal transfer function for two-dimensional reproduction and all resulting properties of the reproduced sound field are termed 2.5-dimensional reproduction, e.g. [11].

In order to simplify the mathematical treatment, we restrict the validity of equations (1)–(6) to our reference line in the target half-plane, i.e. $z = 0$ and $y = y_{\text{ref}}$.

Equation (6) is then given by

$$\tilde{D}(k_x, \omega) = \frac{\tilde{P}(k_x, y_{\text{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)}. \quad (7)$$

Performing an inverse Fourier transform with respect to k_x on (7) yields the driving function $D(x, \omega)$ in temporal spectrum domain as

$$D(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\tilde{P}(k_x, y_{\text{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)} e^{-jk_x x} dk_x. \quad (8)$$

In order that $D(x, \omega)$ is defined, $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ may not exhibit zeros.

From (6) and (8) it is obvious that the driving function is essentially yielded by a division in a spectral domain. We therefore term the presented method *spectral division method*. We emphasize that the spectral division method is not restricted to linear secondary source distributions (refer to Sec. 1).

For convenience, we want to reproduce a virtual plane wave which propagates along the x - y -plane. Recall that we reference the reproduced sound field to the line given by $z = 0$ and $y = y_{\text{ref}} > 0$. Refer to Fig. 2.

$\tilde{P}(k_x, y = y_{\text{ref}}, z = 0, \omega)$ of a monochromatic plane wave of radial frequency $\omega_{\text{pw}} = 2\pi f_{\text{pw}}$ and with unit amplitude propagating in direction $(\theta_{\text{pw}}, \frac{\pi}{2})$ is given by [5]

$$\tilde{P}(k_x, y_{\text{ref}}, 0, \omega) = 2\pi\delta(k_x - k_{\text{pw},x}) e^{-jk_{\text{pw},y} y_{\text{ref}}} \times 2\pi\delta(\omega - \omega_{\text{pw}}), \quad (9)$$

where $k_{\text{pw},x} = k_{\text{pw}} \cos \theta_{\text{pw}}$ and $k_{\text{pw},y} = k_{\text{pw}} \sin \theta_{\text{pw}}$.

Introducing (9) and the reference line into (8) and exploiting the sifting property of the Dirac delta function [10] yields the driving signal $D(x, \omega)$ for the secondary source at position $[x \ 0 \ 0]^T$ as

$$D(x, \omega) = \frac{e^{-jk_{\text{pw},y} y_{\text{ref}}} e^{-jk_{\text{pw},x} x}}{\tilde{G}(k_{\text{pw},x}, y_{\text{ref}}, 0, \omega_{\text{pw}})}. \quad (10)$$

If we thus aim at reproducing a virtual plane wave, we can relax the requirement that $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ may not exhibit zeros as set in (8). From (10) we can deduce that for the reproduction of a plane wave, it is sufficient that $\tilde{G}(k_{\text{pw},x}, y_{\text{ref}}, 0, \omega_{\text{pw}})$ does not exhibit zeros. Arbitrary complex wave fields can be described by a continuum of plane waves [8] so that in this case the strict requirement as dictated by (8) applies.

If the latter requirement is not fulfilled in practical situations, regularization can be applied in order to ensure a good behavior of the inverse of $\tilde{G}(\cdot)$. In the presented approach, the regularization can be applied on individual spatial frequencies k_x which results generally in a more gentle regularization than regularizing the entire inverse problem like in [1].

3. COMPARISON TO CONVENTIONAL APPROACHES

As stated in Sec. 1, analytical sound field reproduction methods like WFS which employ linear distributions of secondary sources assume that the latter are omnidirectional, e.g. [11]. However, real-world loudspeakers are omnidirectional only for low frequencies, but complex radiation patterns evolve for frequencies typically above a few thousand Hertz. In order to assess the benefit of the proposed method, we have to investigate what happens in real-live scenarios when conventional methods are employed, i.e. when an array composed of non-omnidirectional loudspeakers with spatio-temporal transfer function $G_0(\cdot)$ is driven with a driving function designed for omnidirectional secondary sources. We denote the spatio-temporal transfer function assumed by the driving function $G_D(\cdot)$. For convenience, we assume again a virtual plane wave to

be reproduced.

In order to obtain the reproduced sound field $P(\mathbf{x}, \omega)$ we insert (9) in (7), the result in (5), perform an inverse Fourier transform with respect to k_x and exploit the sifting property of the Dirac delta function [10]. $P(\mathbf{x}, \omega)$ is then given by

$$P(\mathbf{x}, \omega) = e^{-jk_{\text{pw},y} y_{\text{ref}}} e^{-jk_{\text{pw},x} x} \frac{\tilde{G}_0(k_{\text{pw},x}, y, z, \omega_{\text{pw}})}{\tilde{G}_D(k_{\text{pw},x}, y_{\text{ref}}, 0, \omega_{\text{pw}})}, \quad (11)$$

In (11), $\tilde{G}_D(k_{\text{pw},x}, y_{\text{ref}}, 0, \omega_{\text{pw}})$ is a single complex number whose absolute value and phase are both dependent on the propagation direction of the virtual plane wave and on the assumed spatio-temporal transfer function of the secondary sources employed.

This means that if the spatio-temporal transfer function of the secondary sources which was assumed in the derivation of the driving function does not match the actual spatio-temporal transfer function of the employed secondary sources, the reproduced sound field will deviate from the desired sound field in amplitude and in phase. Both the amplitude and the phase deviations are dependent on the propagation direction of the virtual plane wave and on the mismatch between actual and assumed secondary source spatio-temporal transfer function.

As a consequence, if such a mismatch is apparent, the reproduced sound field varies in amplitude and phase for different propagation angles. Furthermore, if the mismatch is dependent on the temporal frequency f (what it does in real-life scenarios, see above), the amplitude and phase of a virtual plane wave of given propagation direction carrying a broadband signal varies with the temporal frequency. Especially the frequency dependent amplitude variations are likely to be audible as a timbral coloration.

It is remarkable that the reproduced sound field will exhibit perfectly plane wave fronts for a given frequency f even if a mismatch is apparent. The amplitude decay of the reproduced sound field is dependent on the spatio-temporal transfer function of the secondary sources employed. Omnidirectional sources lead to an amplitude decay for around 3 dB for each doubling of the distance to the secondary source distribution [9]. For secondary sources with a strong focus in a given direction the amplitude decay is lower.

4. RESULTS

In this section, we present simulations of a sample scenario in order to illustrate the theoretical derivations outlined in Sec. 2. We will simulate the sound field of a distribution of secondary sources with given directivity and compare the result with a distribution of omnidirectional secondary sources in order to illustrate the basic properties.

The distribution of non-omnidirectional sources is assumed to be composed of secondary sources with a spatio-temporal transfer function $G_{\text{dipole}}(\cdot)$ given by

$$G_{\text{dipole}}(\mathbf{x}, \omega) = h_1^{(2)}\left(\frac{\omega}{c} r\right) \left(e^{-j\frac{\pi}{6}} Y_1^1(\alpha, \beta) - e^{j\frac{\pi}{6}} Y_1^{-1}(\alpha, \beta) \right), \quad (12)$$

whereby $h_1^{(2)}(\cdot)$ denotes the 1st-order spherical Hankel function of second kind, and $Y_n^m(\cdot)$ the n -th degree, m -th order spherical harmonic [8]. Refer to Fig. 1 for the coordinate system.

$G_{\text{dipole}}(\cdot)$ represents a dipole [8] whose main axis lies in the horizontal plane at an angle of 30° to the x -axis. The normalized far-field directivity of $G_{\text{dipole}}(\cdot)$ is depicted Fig. 3.

For convenience, we apply a numerical Fourier transform in order to obtain $\tilde{G}_{\text{dipole}}(\cdot)$ since an analytic treatment is not straightforward.

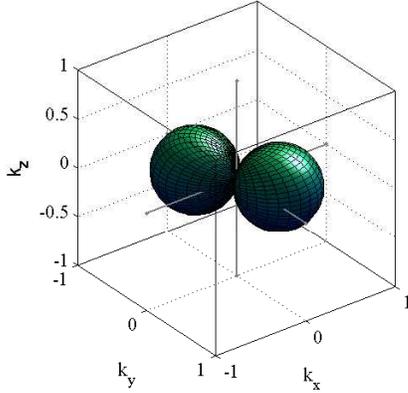


Fig. 3. Normalized far-field directivity of the secondary sources employed in Fig. 4(b).

Like in Sec. 2, we aim at reproducing a virtual plane wave with unit amplitude and propagation direction $(\frac{\pi}{4}, \frac{\pi}{2})$.

It can be shown that $\tilde{G}_{\text{dipole}}(k_{\text{pw},x}, y_{\text{ref}}, 0, \omega_{\text{pw}})$ does not exhibit zeros. It is therefore justified to apply presented approach.

The real part of the sound field reproduced a continuous distribution of such dipoles when it is driven with the presented approach is depicted in Fig. 4(b). The classical situation, i.e. a linear distribution of secondary monopoles, is depicted in Fig. 4(a) for comparison. We reproduced sound fields are very similar inside the target half-

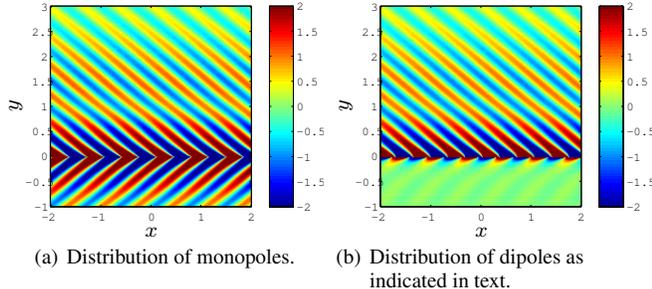


Fig. 4. Sound fields in the horizontal plane reproduced by continuous distributions of secondary sources. Desired sound field is a monochromatic plane wave of frequency $f_{\text{pw}} = 700$ Hz with unit amplitude and propagation direction $(\frac{\pi}{4}, \frac{\pi}{2})$.

plane. For the distribution of monopoles, the sound field reproduced in the other half-space is a perfect mirrored copy of the target half-space. For the distribution of dipoles, the sound field reproduced in the other half-space differs from the perfect mirrored copy with respect to amplitude and phase. The wave fronts are perfectly plane in both half-planes. The observations are in accordance with the discussion carried out in Sec. 3.

5. CONCLUSIONS AND OUTLOOK

The spectral division method applied on linear distributions of secondary sources was presented. The focus of this paper was on the incorporation of the spatio-temporal characteristics of the employed

secondary source and on the properties of the reproduced sound field. It was shown that the spatio-temporal transfer function of the employed secondary sources may not exhibit zeros in the wavenumber domain. Otherwise regularization has to be applied in order to ensure a good behavior of the inverse of the transfer function.

Unlike MIMO approaches where the radiation characteristics of the entire secondary source setup have to be measured in anechoic conditions, it is sufficient in the presented approach to measure only one single secondary source. At this stage it can not be judged whether it is favorable to employ a linear microphone array in the measurement of the secondary source's transfer function or whether a spherical or circular array is advantageous when an appropriate plane wave representation of the measured signals is used.

The treatment of discrete secondary source distributions was beyond the scope of the paper and is subject to future work. Discrete distributions of non-omnidirectional have remarkable properties with respect to spatial aliasing and even provide the potential to suppress spatial aliasing in special situations.

Although not explicitly investigated either in the present paper, it can be shown that the amplitude deviations of the reproduced sound field from perfect reproduction are lower when secondary sources with a with a strong focus in a given direction are employed. It might therefore be favorable to use such loudspeakers especially in large arrays in order to achieve a more balanced amplitude distribution. This circumstance is also subject to future work.

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