Reproduction of Focused Sources by the Spectral Division Method

Sascha Spors and Jens Ahrens
Deutsche Telekom Laboratories, Technische Universität Berlin
Ernst-Reuter-Platz 7, 10587 Berlin, Germany
Email: {sascha.spors,jens.ahrens}@telekom.de

***Abstract*—Sound reproduction methods based on the physical resynthesis of a desired field using loudspeaker arrays are well-established nowadays. Their physical basis allows to resynthesize almost any desired wave field, even the field of sound sources positioned in between the loudspeakers and the listener. Such sources are known as focused sources. This paper will present a novel approach to the reproduction of focused sources with linear loudspeaker arrays. Its formulation is based on a representation of the respective fields in the spatio-temporal frequency domain. The derivation of the loudspeaker driving function is discussed, as well as a number of practical limits, the role of evanescent contributions and the connections to other established techniques.**

I. Introduction

The accurate resynthesis of a sound field using loudspeaker arrays has been a quite active area of research in the last decades. Well known approaches in this context are wave field synthesis (WFS) [1], higher-order Ambisonics (HOA) [2] and a number of least-squares approaches e. g. [3]. Recently, the authors have proposed a novel technique, the spectral division method (SDM) [4], [5].

An interesting property of these approaches is that they principally allow to reproduce the wave field of a source which is positioned in between the loudspeakers and the listener. These are known as *focused sources*, due to their strong relation to acoustic focusing.

Acoustic focusing refers to a variety of techniques to focus acoustic wave fields. These have been developed in diverse application areas like e. g. material analysis or medicine [6]–[8]. The basic concept underlying most of the techniques is the principle of time-delay law focusing or more generally of time-reversal acoustic focusing. Typically, a concentration of acoustic energy at the focus point is desired. For sound reproduction, the goal is to create the illusion of an acoustic source that is situated in front of the loudspeaker array. Note, that this condition implies an important constraint in comparison to the traditional time-reversal principle. Only contributions emerging from the desired focused source should be reproduced at the listener position in order not to confuse the auditory impression by other contributions. Time reversal techniques may result in additional contributions, especially for curved or closed loudspeaker arrays.

So far, the SDM has been applied to the reproduction of plane waves with planar or linear loudspeaker arrays [4], [5]. This paper extends the SDM for the reproduction of focused sources

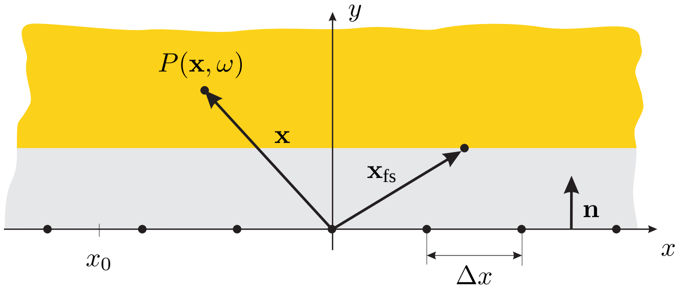


Fig. 1. Geometry underlying the reproduction of focused sources with a linear distribution of secondary point sources. The yellow area denotes the listening area.

using linear loudspeaker arrays. It is organized as follows: Section II outlines the SDM, while Section III extends it towards the reproduction of focused sources. Section IV shows results, discusses the effects of spatial sampling and truncation, and illustrates the links to WFS. The paper is summarized and some conclusions are drawn in Section VI.

II. The Spectral Division Method

The spectral division method, presented in [4], [5], utilizes a formulation of the sound reproduction problem in the Fourier domain. It is applicable to planar and linear loudspeaker arrays. In this paper the considerations are limited to the case of linear loudspeaker arrays due to their practical relevance in sound reproduction. The following section outlines the basic theory.

A. Basic Concept

Without loss of generality, the geometry depicted in Fig. 1 will be assumed for the following considerations. An appropriately driven continuous distribution of monopole sources (secondary sources) is located along the x-axis. The reproduced wave field $P(\mathbf{x},\omega)$ is given as

$$P(\mathbf{x},\omega) = \int_{-\infty}^{\infty} D(\mathbf{x}_0,\omega) G(\mathbf{x}-\mathbf{x}_0,\omega) dx_0 \ , \tag{1}$$

where $\omega = 2\pi f$ denotes the angular frequency, $\mathbf{x} = [x \ y \ z]^T$ an arbitrary position in space and $\mathbf{x}_0 = [x_0 \ \ 0 \ \ 0]^T$ a position on the secondary source distribution. The wave field of the secondary sources is denoted as $G(\mathbf{x}-\mathbf{x}_0,\omega)$, and their weights (driving function) by $D(\mathbf{x}_0,\omega)$. For the sake of simplicity it is assumed that the listeners ears are located in

the upper half-plane ($y > 0$) of the plane $z = 0$. Please refer to [5] for a generalization. The z-coordinate will be discarded in the remainder of this paper.
Equation (1) essentially constitutes a spatial convolution of the driving function with the field of the secondary sources along the x-axis. Hence, the convolution theorem [9] of the Fourier transformation can be utilized. Applying a spatial Fourier transformation with respect to the x-coordinate to Eq. (1) yields

$$\tilde{P}(k_x, y, \omega) = \tilde{D}(k_x, \omega)\, \tilde{G}(k_x, y, \omega)\ , \tag{2}$$

where k_x denotes the wavenumber in x-direction. Quantities in the spatial Fourier domain are indicated by a tilde. Equation (2) can now be solved easily with respect to the driving function $\tilde{D}(k_x, \omega)$ when the spectra of the desired wave field $\tilde{P}(k_x, y, \omega)$ and the secondary sources $\tilde{G}(k_x, y, \omega)$ are known. The former is derived in the following section.

B. Secondary Source Model

Although the SDM allows for the employment of secondary sources with complex radiation characteristics we assume a point source model for simplicity. Loudspeakers with closed cabinets approximately have the properties of a point source. Hence, the point source is a practical model for the secondary sources. The associated transfer function is given as [10]

$$G(\mathbf{x} - \mathbf{x}_0, \omega) = \frac{1}{4\pi} \frac{e^{-j\frac{\omega}{c}|\mathbf{x}-\mathbf{x}_0|}}{|\mathbf{x} - \mathbf{x}_0|}\ , \tag{3}$$

where c denotes the speed of sound. The Fourier transformation with respect to x can be derived from [11] as

$$\tilde{G}(k_x, y, \omega) = \begin{cases} -\frac{j}{4} H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2}\ y\right) & ,\ |k_x| < \left|\frac{\omega}{c}\right| \\ \frac{1}{2\pi} K_0\left(\sqrt{k_x^2 - (\frac{\omega}{c})^2}\ y\right) & ,\ \left|\frac{\omega}{c}\right| < |k_x| \end{cases}\ , \tag{4}$$

where $H_0^{(2)}(\cdot)$ denotes the zero-th order Hankel function of second kind and $K_0(\cdot)$ the zero-th order modified Bessel function of second kind [12]. Note that (4) is valid only for $y > 0$. The spectrum of the secondary sources (4) consists of two parts: a traveling contribution for $|k_x| < \left|\frac{\omega}{c}\right|$ and an evanescent contribution for $\left|\frac{\omega}{c}\right| < |k_x|$.
Evanescent waves are waves which exhibit no phase variation in at least one spatial dimension and decay exponentially in these directions [10]. They emerge from solutions of the acoustic wave equation with exhibit at least one imaginary wave number.

C. 2.5-dimensional Reproduction

From a physical point of view, the natural choice for the characteristics of secondary sources used for two-dimensional reproduction would be the elementary solution of the wave equation in two dimensions. The resulting transfer function is given by the two-dimensional free-field Greens function, which can be interpreted as the field produced by a line source. Using point sources as secondary sources for the reproduction in a plane results in a dimensionality mismatch, therefore such methods are often termed as *2.5-dimensional reproduction*.
It is well known from other reproduction techniques, e. g. WFS and HOA, that 2.5-dimensional reproduction techniques suffer from artifacts [13], [14]. Most prominent are amplitude and spectral errors in this context. For the SDM these artifacts have been analyzed in detail for the reproduction of plane waves [5].

III. Focused Sources using the SDM

So far the spectral division method, as outlined in Section II, has been applied to the reproduction of plane waves. This section presents the extension to focused sources.

A. Model of Focused Source

In order to derive the driving function, a model for the desired wave field $P(\mathbf{x}, \omega)$ is needed. A point source placed within the listening area is not a suitable model in this context. The resulting solution would have to violate causality below the focus point, since the secondary sources can only emit a wave field which travels towards the focus point. A suitable model for a focused source can be formulated by prescribing the field above the focus point as a point source. The impression of a source within the listening area is hence only conveyed for focus points located in between the listener and the secondary sources. However, this is a well known limitation in the context of auralization. The resulting listening area is indicated by the yellow area in Fig. 1.
Modeling an acoustic point source above the focus point $y > y_{\text{fs}}$ yields

$$P_{\text{fs}}(\mathbf{x}, \omega) = \hat{P}_{\text{fs}}(\omega)\ \frac{1}{4\pi} \frac{e^{-j\frac{\omega}{c}|\mathbf{x}-\mathbf{x}_{\text{fs}}|}}{|\mathbf{x} - \mathbf{x}_{\text{fs}}|},\ \text{for}\ y > y_{\text{fs}} > 0\ , \tag{5}$$

where $\mathbf{x}_{\text{fs}} = [x_{\text{fs}}\ \ y_{\text{fs}}]^T$ denotes the position of the focused source (focus point) and $\hat{P}_{\text{fs}}(\omega)$ the temporal spectrum of the focused source. Based on the model (5), the driving function is derived in the next section.

B. Derivation of Driving Function

The spatial spectrum $\tilde{P}_{\text{fs}}(k_x, y, \omega)$ of the focused source (5) is required in order to analytically derive the driving function by application of (2). The spatio-temporal spectrum can be derived from (4) by applying the shift-theorem [9] of the Fourier transformation. It is given as

$$\tilde{P}_{\text{fs}}(k_x, y, \omega) = \hat{P}_{\text{fs}}(\omega)\ e^{jk_x x_{\text{fs}}} \times \\ \times \begin{cases} -\frac{j}{4} H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2}\ (y - y_{\text{fs}})\right) & ,\ |k_x| < \left|\frac{\omega}{c}\right| \\ \frac{1}{2\pi} K_0\left(\sqrt{k_x^2 - (\frac{\omega}{c})^2}\ (y - y_{\text{fs}})\right) & ,\ \left|\frac{\omega}{c}\right| < |k_x| \end{cases}\ , \tag{6}$$

which is valid for $y > y_{\text{fs}} > 0$. The driving function is given by a division of the spectrum of the desired field $\tilde{P}_{\text{fs}}(k_x, y, \omega)$ and the spectrum of the secondary sources $\tilde{G}(k_x, y, \omega)$

$$\tilde{D}_{\text{fs}}(k_x, \omega) = \hat{P}_{\text{fs}}(\omega)\ e^{jk_x x_{\text{fs}}} \times \\ \times \begin{cases} -\frac{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2}\ (y - y_{\text{fs}})\right)}{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2} y\right)} & ,\ |k_x| < \left|\frac{\omega}{c}\right| \\ \frac{K_0\left(\sqrt{k_x^2 - (\frac{\omega}{c})^2}\ (y - y_{\text{fs}})\right)}{K_0\left(\sqrt{k_x^2 - (\frac{\omega}{c})^2} y\right)} & ,\ \left|\frac{\omega}{c}\right| < |k_x| \end{cases}\ . \tag{7}$$

It is easy to conclude from Eq. (7) that the driving function depends on the (listener) distance y to the secondary source distribution. This is a property of 2.5-dimensional reproduction. The wave field can only be reproduced correctly on a reference line $y = y_{\text{ref}}$ parallel to the secondary source distribution [5]. In the remainder of this paper, y will be replaced by y_{ref} whenever the driving function (7) is used.

Even though we are only matching the spectra of the reproduced and the desired wave field for $y > y_{\text{fs}}$, the wave field for $0 < y < y_{\text{fs}}$ will be determined uniquely. This follows from the Rayleigh integrals [10]. It is nevertheless straightforward to repeat the steps above for the region $y < y_{\text{fs}}$, in order to derive an alternative formulation of the driving function.

Note, the driving function is not defined for $y_{\text{ref}} - y_{\text{fs}} = 0$ due to the properties of the involved functions. The driving function (7) is composed of a propagating and an evanescent part. The latter is subject to excessive levels in the driving function, as will be discussed in the following.

C. Near-Field Contributions

Using large argument approximations of the modified Bessel function and the Hankel function [12] yields the following approximation of the spectrum of the driving function (7)

$$\tilde{D}_{\text{fs}}(k_x, \omega) \approx \hat{P}_{\text{fs}}(\omega)\, e^{jk_x x_{\text{fs}}} \sqrt{\frac{y_{\text{ref}}}{y_{\text{ref}} - y_{\text{fs}}}} \times \\ \times \begin{cases} e^{-j\sqrt{(\frac{\omega}{c})^2 - k_x^2}\, y_{\text{fs}}} & , \ |k_x| < \left|\frac{\omega}{c}\right| \\ e^{\sqrt{k_x^2 - (\frac{\omega}{c})^2}\, y_{\text{fs}}} & , \ \left|\frac{\omega}{c}\right| < |k_x| \end{cases} . \tag{8}$$

The approximation given by (8) holds for

$$\sqrt{\left|(\frac{\omega}{c})^2 - k_x^2\right|}\; y_{\text{ref}} \gg 1 \text{ , and} \\ \sqrt{\left|(\frac{\omega}{c})^2 - k_x^2\right|}\; (y_{\text{ref}} - y_{\text{fs}}) \gg 1 \ .$$

Hence, for large distances of the focus point to the secondary source distribution ($y_{\text{fs}} \gg 1$) and for large distances of the reference line to the focus point (($y_{\text{ref}} - y_{\text{fs}}) \gg 1$), or for high temporal/spatial frequencies in case of the propagating/evanescent part. The approximation has shown to be quite accurate for typical frequencies (above some hundert Herz) and distances (bigger than some ten centimeters) used in audio reproduction.

It is evident from (8) that the evanescent part of the driving function becomes very large for focused sources far away from the secondary source distribution and for high spatial or low temporal frequencies. This can be concluded intuitively when considering the exponentially decay of the evanescent contributions of the individual secondary sources with distance to the secondary source distribution.

D. Modified Driving Function

A solution to overcome the potential problems with the evanescent contributions in the driving function (7) is to model the focused source without these. As first approach, the evanescent contributions in (6) are simply neglected. This results in the following modified driving function

$$\tilde{D}_{\text{mod,fs}}(k_x, \omega) = \hat{P}_{\text{fs}}(\omega)\, e^{jk_x x_{\text{fs}}} \times \\ \times \begin{cases} -\frac{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2}\ (y_{\text{ref}} - y_{\text{fs}})\right)}{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2}\ y_{\text{ref}}\right)} & , \ |k_x| < \left|\frac{\omega}{c}\right| \\ 0 & , \ \left|\frac{\omega}{c}\right| < |k_x| \end{cases} . \tag{9}$$

As a consequence, the evanescent part of the focused source will not be resynthesized. However, this modification results in a more stable driving function, as will be shown in the next section. To the knowledge of the authors, the role of evanescent contributions in human perception seems to be unclear at the current state of research.

IV. Results

The following section discusses the properties of the proposed approach and illustrates its relations to WFS.

A. Reproduced Wave Field

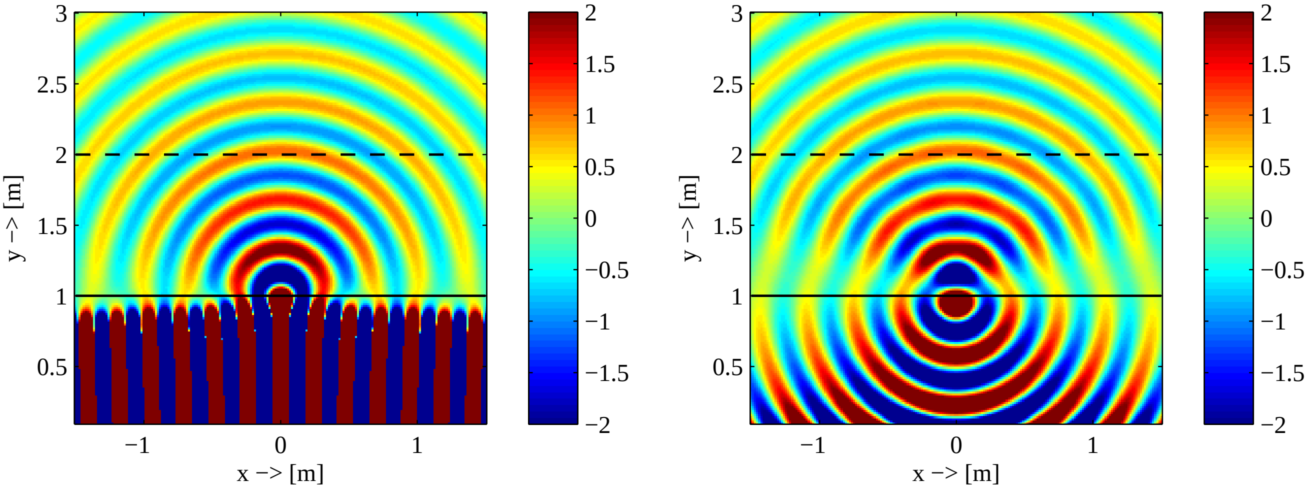
The reproduced wave field is given by introducing the driving function $\tilde{D}_{\text{fs}}(k_x, \omega)$ together with the spectrum of the secondary sources into (2). Inverse Fourier transformation yields then the reproduced wave field in the temporal frequency domain. Performing these steps for a position on the reference line $y = y_{\text{ref}}$ yields that the desired wave field is indeed reconstructed perfectly there. In order to illustrate the properties of the reproduced wave field at other positions numerical simulations of (2) and the inverse spatial Fourier transformation have been performed.

Fig. 2(a) shows the reproduced wave field for the driving function $\tilde{D}_{\text{fs}}(k_x, \omega)$ including the evanescent contributions (7) and Fig. 2(b) for the modified driving function $\tilde{D}_{\text{mod,fs}}(k_x, \omega)$ excluding the evanescent contributions (9). The reproduction of a focused source at position $\mathbf{x}_{\text{fs}} = [0\ 1]^T$ m emitting a monochromatic signal with $f_s = 1000$ Hz using a continuous secondary source distribution is considered. The reference line is chosen to $y_{\text{ref}} = 2$ m (dashed line in Fig. 2).

As already predicted in Section III-C and clearly visible in Fig. 2(a), the driving function (7) including the evanescent contributions produces excessive amplitudes in the reproduced wave field below the focus point ($y < y_{\text{fs}}$). Above the focus point ($y > y_{\text{fs}}$), the field is resynthesized without visible artifacts. However, these excessive contributions will render this approach unfeasible in practice.

The modified driving function (9), excluding the evanescent contributions used in Fig. 2(b) does not show these problems. The amplitudes are bounded to reasonable levels. However the hard truncation proposed in Section III-D, seems to cause artifacts in the reproduced wave field above the focus point ($y > y_{\text{fs}}$). The amplitude of the wave fronts shows some deviations from the desired field.

A more in-depth analysis of the reproduced wave fields shown in Fig. 2 revealed that the amplitude decay with distance to the secondary source distribution of the focused source does not equal the decay of a point source placed at the focus point. This is a well known artifact of 2.5-dimensional reproduction. However, in the case of focused sources the amplitude decay



(a) Driving function (7) including evanescent contributions

(b) Modified driving function (9) excluding evanescent contributions

Fig. 2. Reproduced wave field for a monochromatic focused source with $f_s = 1000$ Hz, $\mathbf{x}_{\text{fs}} = [0\ 1]^T$ m and $y_{\text{ref}} = 2$ m using a continuous secondary source distribution. The level is normalized to the reference line, values are clipped.

seems to differ not so much as for the reproduction of plane waves. The detailed analysis of artifacts and the improvement is subject to future work.

Practical implementations of the proposed approach will use a finite number of loudspeakers placed at discrete positions. This implies a spatial sampling and truncation process that is discussed in the following two subsections.

B. Spatial Sampling

The discretization of the secondary source distribution is modeled by spatial sampling of the driving function. This is performed by multiplying $D_{\text{fs}}(x,\omega)$ with a series of spatial Dirac functions at the positions of the loudspeakers. For an equidistant spacing this reads

$$D_{fs,S}(x,\omega) = D_{\text{fs}}(x,\omega) \cdot \frac{1}{\Delta x} \sum_{\mu=-\infty}^{\infty} \delta(x - \Delta x \mu) \ , \qquad (10)$$

where $D_{fs,S}(x,\omega)$ denotes the sampled driving function and Δx the distance (sampling period) between the sampling positions (indicated by the dots • in Fig. 1). Applying a spatial Fourier transformation to (10) results in

$$\tilde{D}_{fs,S}(k_x,\omega) = 2\pi \sum_{\eta=-\infty}^{\infty} \tilde{D}_{\text{fs}}\left(k_x - \frac{2\pi}{\Delta x}\eta, \omega\right) \ . \qquad (11)$$

Equation (11) states that the spectrum $\tilde{D}_{fs,S}(k_x,\omega)$ of the sampled driving function is given as a superposition of the shifted continuous spectra $\tilde{D}_{\text{fs}}(k_x - \frac{2\pi}{\Delta x}\eta,\omega)$ of the driving function. Introducing the spectrum of the sampled driving function $\tilde{D}_{fs,S}(k_x,\omega)$ into (2) results in the spectrum $\tilde{P}_{fs,S}(k_x,y,\omega)$ of the wave field reproduced by a spatially discrete secondary source distribution.

Above considerations can be used to qualitatively and quantitatively discuss the effects of spatial sampling for focused sources. The same methodology as outlined above has been used for a detailed analysis of sampling artifacts for WFS [15], [16]. It is straightforward to apply these methods also here.

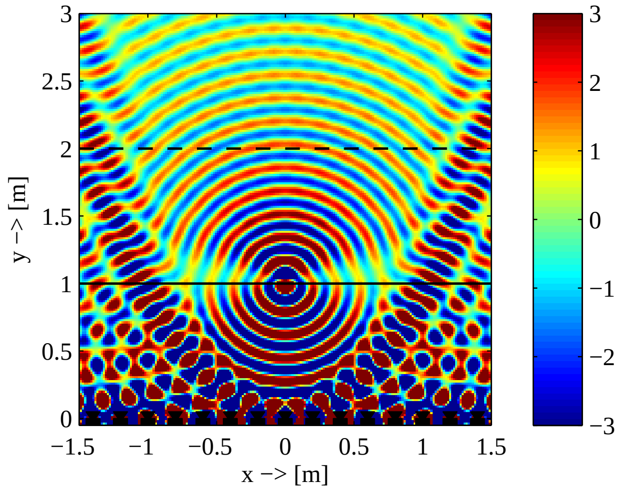


Fig. 3. Wave field reproduced for a monochromatic focused source using $\tilde{D}_{\text{mod,fs}}(k_x,\omega)$ with $f_s = 2000$ Hz, $\mathbf{x}_{\text{fs}} = [0\ 1]$ m and $y_{\text{ref}} = 2$ m by a discrete secondary source distribution with $\Delta x = 0.20$ m. The level is normalized to the reference line, values are clipped.

Fig. 3 shows a numerical simulation of the wave field reproduced by a spatially discrete secondary source distribution with $\Delta x = 0.20$ m using (11) and the modified driving function $\tilde{D}_{\text{mod,fs}}(k_x,\omega)$. The focused source emits a monochromatic signal with $f_s = 2000$ Hz. Sampling artifacts are clearly visible in the reproduced wave field, especially close to the secondary sources. However, no sampling artifacts are visible in the vicinity of the focus point. This is an interesting property of focused sources that has been observed also in the context of WFS [16]. An in-depth analysis is subject to future research.

C. Truncation

Practical implementations will not only be realized by spatially discrete distribution of individual secondary sources but will also be of finite length. This constitutes a truncation of the secondary source distribution. Mathematically, truncation can be modeled by multiplying the secondary source driving function $D_{\text{fs}}(x,\omega)$ with a suitable window function $w(x)$. Incorporating $w(x)$ into (1) yields the wave field $P_{\text{fs,tr}}(\mathbf{x},\omega)$ reproduced by a truncated linear array [5].

Quantitatively, truncation will have two consequences: (1) a limited listening area and (2) an enlarged focus point. The effective listening area can be approximated quite well by geometric means. It is given by the area in front of the loudspeaker array which is bounded by lines through the focus point passing the secondary source distribution at its ends in a tangent like manner [16]. Severe truncation artifacts are present outside this area and some minor deviations are present in the listening area. The second consequence of truncation is well known from optics and there often referred to as diffraction limited system.

V. Comparison with Wave Field Synthesis

Focused sources are a basic feature of WFS [13]. The links between the proposed approach and acoustic focusing by WFS are of special interest, since the properties of these have been investigated in quite some detail for the latter [16].

Qualitatively, the link can be established by comparing the approximation (8) of the spatial spectrum of the driving function with the spectrum of the WFS driving function derived in [16, eq.(10)]. Both are equal besides a normalization factor. The link to the traditional driving function for WFS [13, eq.(2.30)] can be established by following the argumentation given in [16]. In summary, performing a far-field approximation of the inverse spatial Fourier transformation of the approximated driving function (8) will result (besides some normalization factors) in the traditional WFS driving function for focused sources. Hence in the context of focused sources, WFS can be regarded as an approximation of the SDM. A similar conclusion was also drawn for the reproduction of plane waves with the spectral division method [5].

As a consequence of the close relationship between the proposed approach and WFS, most of the properties that have been reported for WFS will hold also here [16]. This holds especially when the approximations used to derive the link are valid, hence for high-frequencies, large distances y_{fs} of the focused source to the secondary source distribution and listener positions y far away from the focused source. Note, that these limits have to be seen in the context of typical wave lengths appearing in audio reproduction.

VI. Conclusions

This paper presents an approach to acoustic focusing using linear loudspeaker arrays which is based upon a formulation of the underlying problem in the spatio-temporal frequency domain. The driving function is derived by spectral division in the spatio-temporal frequency domain and inverse Fourier transformation. It was shown that the evanescent contributions of the desired focused source cannot be reconstructed without accepting high levels in the driving signals of the secondary sources. Hence, in practice it is favorable to neglect the evanescent contributions of the focused source. However, the psychoacoustic implications seem to be unclear at the current stage.

The derived results in conjunction with the results in [16] further indicate that the artifacts of 2.5D reproduction are different for focused sources than for plane waves. Interestingly, acoustic focusing as used currently in WFS can be regarded as an approximation of the presented approach. Hence, the presented approach will have benefits for focused sources placed close to the secondary source distribution and for low frequencies. Furthermore, extensions to the spectral division method like explicit consideration of the secondary source directivity can be applied straightforwardly.

The presented approach can also be applied to other application areas like e. g. ultrasonic imaging. Future research will include a more detailed analysis of the physical and psychoanalytical properties of the presented approach.

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