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On the Scattering of Synthetic Sound Fields

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ABSTRACT
In sound field synthesis a given arrangement of elementary sound sources is employed in order to synthesize a sound field with given desired physical properties over an extended region. The calculation of the driving signals of these secondary sources typically assumes free-field propagation conditions. The present paper investigates the scattering of such synthetic sound fields from unavoidable scattering objects like the head and body of a person apparent in the target region. It is shown that the basic mechanisms are similar to the scattering of natural sound fields. Though, synthetic sound fields can exhibit properties different to those of natural sound fields. Consequently, in such cases also the scattered synthetic sound fields exhibit properties different to those of scattered natural sound fields.

1. INTRODUCTION

Wave Field Synthesis (WFS) [1], Near-field Compensated Higher Order Ambisonics (NFC-HOA) [2], and the Spectral Division Method (SDM) [3] are the three best known analytical methods of sound field synthesis. WFS bases on the Rayleigh integrals or the Kirchhoff-Helmholtz integral, respectively. It constitutes an implicit solution to the underlying physical problem. NFC-HOA and SDM employ explicit solutions of the underlying physical problem by solving the synthesis equation in a transformed domain.

At first stage, all above mentioned methods assume a continuous distribution of elementary sound sources termed "secondary sources". These secondary sources are driven such that their emitted sound fields superpose and make up a sound field with given desired physical properties. In theory, any source-free sound field can be synthesized inside the secondary source distribution if the latter encloses the receiver volume.

The calculation of the driving signals typically assumes free-field conditions, i.e. it assumes that no objects are apparent in the target area which influence the propagation of the sound fields emitted by the secondary sources. Though, when a person listens to the synthesized sound field the latter is distorted. In the present paper, we present an analysis of the scattering of such synthetic sound fields from objects which are apparent in the target area. We compare the results to the scattering of natural sound fields. Spherical secondary source distributions will be treated as an example for three-dimensional synthesis and circular distributions as an example for 2½-dimensional synthesis [4]. The qualitative properties of all other geometries of secondary source distributions can be deduced from the presented results.

The above mentioned requirement of a continuous distribution of secondary sources can not be implemented in practice with today's available technology. It is rather such that arrangements of spatially discrete loudspeakers have to be used. As a consequence, a number of distortions of the spatial structure of the synthesized sound field arise above a given frequency. These artifacts are commonly referred to as *spatial aliasing* [5].

Obviously, when such a discrete secondary source distribution is considered, not only the desired component of the synthetic sound field is scattered but also the additional artifacts which arise due to spatial discretization. This circumstance will also be investigated in the presented paper.

2. NOMENCLATURE AND MATHEMATICAL PRELIMINARIES

In the remainder of the paper, we use the following notational conventions: For scalar variables upper case denotes time-frequency domain. Vectors are denoted by lower case boldface. The three-dimensional position vector in Cartesian coordinates is given by $\mathbf{x} = [x\ y\ z]^T$. The Cartesian coordinates are linked to the spherical coordinates via $x = r\cos\alpha\sin\beta$, $y = r\sin\alpha\sin\beta$, and $z = r\cos\beta$, whereby α denotes the azimuth, β the zenith angle. The coordinate system is depicted in Fig. 1.

The acoustic wavenumber is denoted by k. It is related to the time frequency f by $k^2 = \left(\frac{\omega}{c}\right)^2$ with $\omega = 2\pi f$ being the radial frequency and c the speed of sound. Outgoing spherical waves are denoted by $\frac{1}{r}e^{-i\frac{\omega}{c}r}$, monochromatic plane waves by $e^{-i\mathbf{k}_{\mathrm{pw}}^T\mathbf{x}}$, with $\mathbf{k}_{\mathrm{pw}}^T = [k_{\mathrm{pw},x}\quad k_{\mathrm{pw},y}\quad k_{\mathrm{pw},z}] = k_{\mathrm{pw}}\cdot[\cos\theta_{\mathrm{pw}}\sin\phi_{\mathrm{pw}},\quad \sin\theta_{\mathrm{pw}}\sin\phi_{\mathrm{pw}},\ \cos\phi_{\mathrm{pw}}]$ and $(\theta_{\mathrm{pw}}, \phi_{\mathrm{pw}})$ denoting the azimuth and zenith angle of the propagation direction of the plane wave. The imaginary unit is denoted by i ($i^2 = -1$).

When free-field propagation in a homogeneous medium is assumed, a sound field $S(\mathbf{x}, \omega)$ which is source-free in the domain of interest can be uniquely described by a series of spherical harmonics expansion coefficients $\mathring{S}_n^m(r,\omega)$ or of coefficients $\breve{S}_n^m(\omega)$ respectively as [6, 7]

$$S(\mathbf{x},\omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \underbrace{\breve{S}_n^m(\omega)\, j_n\left(\frac{\omega}{c}r\right)}_{=\ \mathring{S}_n^m(r,\omega)} Y_n^m(\beta,\alpha)\ , \quad (1)$$

whereby $j_n(\cdot)$ denotes the n-th order spherical Bessel function of first kind and $Y_n^m(\cdot)$ the surface spherical harmonics [6].

Sound fields radiating away from the coordinate origin can be represented by [6, 7]

$$S(\mathbf{x},\omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \hat{S}_n^m(\omega)\, h_n^{(2)}\left(\frac{\omega}{c}r\right) Y_n^m(\beta,\alpha)\ , \quad (2)$$

with $h_n^{(2)}(\cdot)$ denoting the spherical Hankel function of second kind.

The spherical harmonics $Y_n^m(\beta,\alpha)$ may be defined as [7]

$$Y_n^m(\beta,\alpha) = (-1)^m\sqrt{\frac{(2n+1)}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}}P_n^{|m|}(\cos\beta)\, e^{im\alpha}\ , \quad (3)$$

with $P_n^{|m|}(\cdot)$ denoting the $|m|$-th order associated Legendre function of n-th degree.

When quantities do not carry further indices, then free-field conditions are assumed. When an incoming sound field $S(\mathbf{x},\omega)$ is scattered at an object, the resulting sound field $S_{\mathrm{total}}(\mathbf{x},\omega)$ is given by the sum of the incoming sound field and the scattered sound field $S_{\mathrm{scat}}(\mathbf{x},\omega)$ as [7]

$$S_{\mathrm{total}}(\mathbf{x},\omega) = S(\mathbf{x},\omega) + S_{\mathrm{scat}}(\mathbf{x},\omega)\ . \quad (4)$$

Assuming an acoustically rigid spherical scattering object of radius A which is centered around the coordinate system, then the coefficients $\hat{S}_{n,\mathrm{scat}}^m(\omega)$ are given by [7, Eq. (4.2.10), p. 146]

$$\hat{S}^m_{n,\text{scat}}(\omega) = -\frac{j_n'\left(\frac{\omega}{c}A\right)}{h_n^{(2)\prime}\left(\frac{\omega}{c}A\right)}\breve{S}^m_n(\omega) \ . \tag{5}$$

The prime in (5) indicates differentiation with respect to the argument.

3. SPHERICAL SECONDARY SOURCE DISTRIBUTIONS

3.1. Analytical Derivation

The sound field $S(\mathbf{x},\omega)$ synthesized by a continuous acoustically transparent spherical secondary source distribution of radius R centered around the coordinate origin is given by [8, 9, 10]

$$S(\mathbf{x},\omega) = \int_0^{2\pi}\int_0^{\pi} D\left(\mathbf{x}_0,\omega\right) \ G\left(\mathbf{x}-\mathbf{x}_0,\omega\right) \ \sin\beta_0 \ d\beta_0 \ d\alpha_0 \ , \tag{6}$$

$D(\mathbf{x}_0,\omega)$ denotes the driving function of the secondary source located at

$$\mathbf{x}_0 = R \ \left[\cos\alpha_0\sin\beta_0 \quad \sin\alpha_0\sin\beta_0 \quad \cos\beta_0\right]^T$$

and $G(\mathbf{x}-\mathbf{x}_0,\omega)$ its spatial transfer function. Refer to Fig. 1 for an illustration of the setup. The following derivation follows [8, 4].

In order for (6) to hold, $G(\mathbf{x}-\mathbf{x}_0,\omega)$ has to be invariant with respect to rotation around the origin of the secondary source distribution [4]. Eq. (6) is generally also referred to as *synthesis equation*.

Eq. (6) can be interpreted as a convolution along the surface of a sphere in which case the convolution theorem

$$\mathring{S}^m_n(r,\omega) = 2\pi R\sqrt{\frac{4\pi}{2n+1}} \ \mathring{D}^m_n(\omega)\cdot\mathring{G}^0_n(r,\omega) \ , \tag{7}$$

applies [11]. For all frequencies ω other than forbidden frequencies [8, 12] (7) may be written as

$$\breve{S}^m_n(\omega) = 2\pi R\sqrt{\frac{4\pi}{2n+1}} \ \mathring{D}^m_n(\omega)\cdot\breve{G}^0_n(\omega) \ . \tag{8}$$

The coefficients $\mathring{D}^m_n(\omega)$ of the driving function for synthesis of a sound field given by $\mathring{S}^m_n(\omega,r)$ or $\breve{S}^m_n(\omega)$ respectively can thus be determined to be

$$\mathring{D}^m_n(\omega) = \frac{1}{2\pi R}\sqrt{\frac{2n+1}{4\pi}} \ \frac{\breve{S}^m_n(\omega)}{\breve{G}^0_n(\omega)} \ . \tag{9}$$

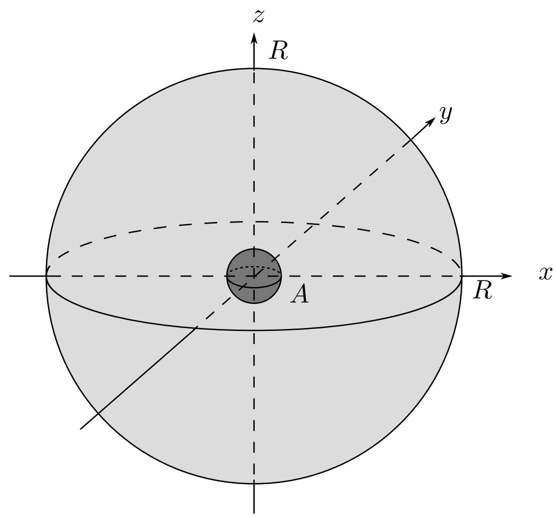


Fig. 1: Spherical secondary source distribution of radius R and spherical scattering object of radius A both centered around the coordinate origin.

The driving function $D\left(\mathbf{x},\omega\right)$ can be composed from the coefficients $\mathring{D}^m_n(\omega)$ as indicated in (1). In order that (9) holds, $\breve{G}^0_n(\omega)$ may not exhibit zeros. This requirement is fulfilled for secondary monopoles under free-field conditions. When (9) is applied, then the desired sound field $S(\mathbf{x},\omega)$ is perfectly synthesized [8, 13].

Now assume an acoustically rigid spherical object of radius $A < R$ which is also centered around the origin of the coordinate system (Fig. 1). This choice of geometry and properties of the scattering object has been made for mathematical simplicity and does not restrict the validity of the following results.

The objective is determining the scattering of the synthetic sound field $S(\mathbf{x},\omega)$ from the scattering object. From (5), we know that the coefficients $\hat{G}^m_{n,\text{scat}}(\omega)$ of the scattered spatial transfer function of the secondary sources are given by

$$\hat{G}^m_{n,\text{scat}}(\omega) = -\frac{j_n'\left(\frac{\omega}{c}A\right)}{h_n^{(2)\prime}\left(\frac{\omega}{c}A\right)}\breve{G}^m_n(\omega) \ . \tag{10}$$

The total sound field $S_{\text{total}}(\mathbf{x},\omega)$ evoked by the spherical secondary source distribution when the scattering object is apparent is the sum of the sound field $S(\mathbf{x},\omega)$ synthesized under free-field conditions and the scattering of $S(\mathbf{x},\omega)$ at the spherical object (Eq. (4)).

The scattered synthetic sound field $S_{\mathrm{scat}}(\mathbf{x},\omega)$ is given by (refer to (6))

$$S_{\mathrm{scat}}(\mathbf{x},\omega) = \int_0^{2\pi}\int_0^{\pi} D\left(\mathbf{x}_0,\omega\right)\; G_{\mathrm{scat}}\left(\mathbf{x}-\mathbf{x}_0,\omega\right)\; \sin\beta_0 \; d\beta_0 \; d\alpha_0 \; . \tag{11}$$

Recall that we assume free-field conditions in the calculation of the driving function $D\left(\mathbf{x}_0,\omega\right)$.

Since $G_{\mathrm{scat}}\left(\mathbf{x}-\mathbf{x}_0,\omega\right)$ is invariant with respect to rotation around the origin of the coordinate system, the convolution theorem (8) still holds. It is stated here again for convenience as

$$\hat{S}^m_{n,\mathrm{scat}}(\omega) = 2\pi R\sqrt{\frac{4\pi}{2n+1}}\; \mathring{D}^m_n(\omega)\cdot \breve{G}^0_{n,\mathrm{scat}}(\omega) \; . \tag{12}$$

Introducing the coefficients $\mathring{D}^m_n(\omega)$ of the free-field driving function given by (9) into (12) shows that the coefficients $\breve{S}^m_{n,\mathrm{scat}}(\omega)$ of the synthesized sound field are given by

$$\hat{S}^m_{n,\mathrm{scat}}(\omega) = -\frac{j_n'\left(\frac{\omega}{c}A\right)}{h_n^{(2)\prime}\left(\frac{\omega}{c}A\right)}\breve{S}^m_n(\omega) \; . \tag{13}$$

Comparing (13) to (5) shows that the scattered synthesized sound field $S_{\mathrm{scat}}(\mathbf{x},\omega)$ does correspond to the desired sound field $S(\mathbf{x},\omega)$ scattered from the scattering object. In other words, the scattering of a sound field is independent of the properties of the sound source which evokes the sound field under consideration. It is therefore inconsequential whether the plane wave we considered is evoked by a sound source at infinite distance or by a secondary source distribution enclosing the domain of interest. This result is also represented by the fact that (5) does not make any assumption on the sound source.

3.2. Simulations

In this section, we present simulations which confirm the results derived in Sec. 3.1. As mentioned in Sec. 1, the continuous secondary source distribution assumed in Sec. 3.1 can not be implemented but discrete arrangements of a finite number of loudspeakers have to be used. This circumstance does not have a considerable impact on the synthesized sound field at low frequencies. The latter situation will be illustrated in the following. At higher frequencies, severe deviations from the desired sound field arise [4]. This situation will be treated in Sec. 4 in conjunction with circular secondary source distributions.

Fig. 2(a) depicts a plane wave of frequency f = 1000 Hz scattered from a spherical object of radius A = 0.3 m. Fig. 2(b) depicts the same scenario whereas the plane wave is synthesized by a spherical distribution of 1568 discrete omnidirectional loudspeakers arranged on a Gaussian sampling scheme [11] with R = 1.5 m. All parameters were chosen such that the visual inspection of the simulations allows for a meaningful interpretation.

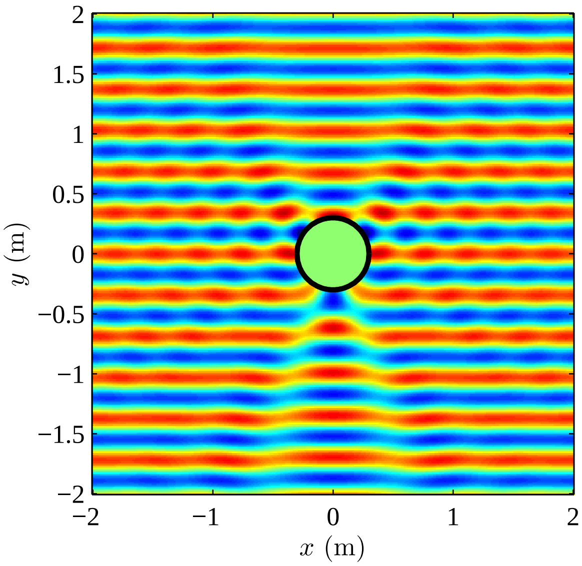
As can be seen from Fig. 2, the scattered sound fields are indeed similar when the region is considered which is inside the secondary source distribution.

4. CIRCULAR SECONDARY SOURCE DISTRIBUTIONS

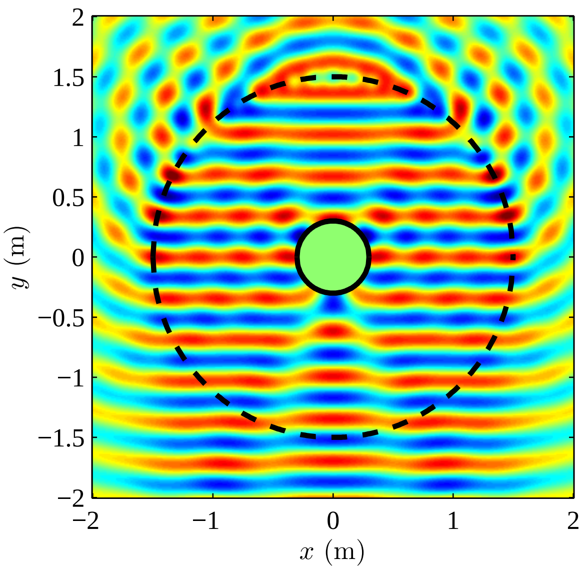
Circular secondary source distributions are representatives of a special category of secondary source distributions since they do not enclose a given target volume. Unlike e.g. the spherical distribution treated in Sec. 3, such non-enclosing distributions are not capable of perfectly synthesizing arbitrary source-free sound fields [4]. Circular secondary source distributions rather aim at sound field synthesis in a plane, typically in the horizontal plane. Such a situation is then termed *2½-dimensional* synthesis since it is neither purely two-dimensional or three-dimensional but constitutes something in between [5].

The most prominent property of 2½-dimensional synthesis is the fact that the amplitude of the synthesized sound field deviates from the desired one [4]. This means that, since the synthesized sound field is different to the desired one, the scattered synthesized sound field is different to the scattered desired sound field.

The analytical treatment presented in Sec. 3 can straightforwardly be adapted to the case of circular spherical distributions. We waive a presentation of the details here since the results are not revealing. We rather provide simulations which illustrate



(a) Scattered plane wave



(b) Scattered synthetic plane wave. The dashed line indicates the secondary source distribution. $R = 1.5$ m.

Fig. 2: Sound fields the horizontal plane. The plane waves are propagating into negative y-direction and carry a monochromatic signal of frequency $f = 1000$ Hz. The radius of the scattering object is $A = 0.3$ m.

the basic properties. A detailed mathematical treatment of sound field synthesis employing circular secondary source distributions can be found in [8, 4].

In the remainder of this section, we assume the geometrical setup depicted in Fig. 3 whereby the secondary source distribution is composed of 56 equiangularly spaced omnidirectional secondary sources driven in order to synthesized a virtual plane wave propagating in negative y-direction.

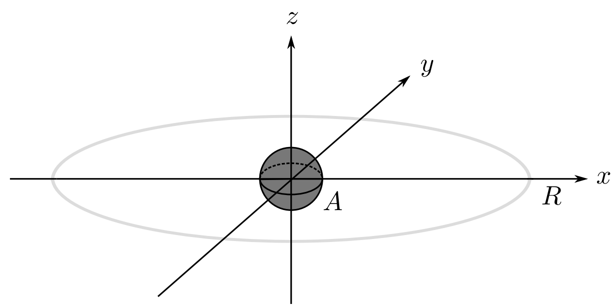


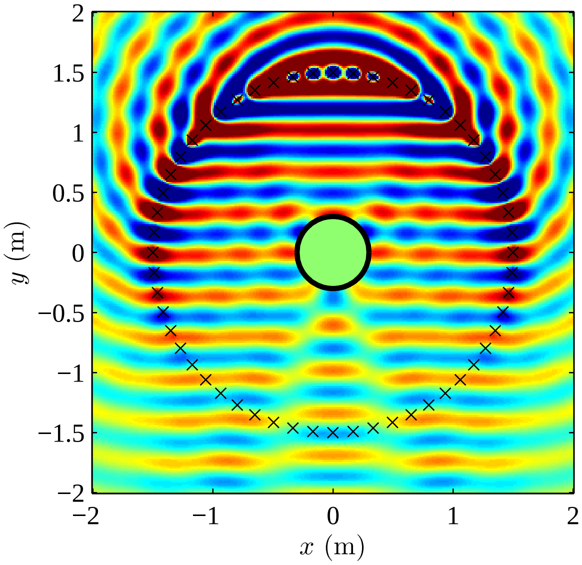
Fig. 3: Circular secondary source distribution of radius R in the horizontal plane and spherical scattering object of radius A. Both objects are centered around the coordinate origin.

Fig. 4(a) depicts the synthesized sound field for a frequency of $f = 1000$ Hz. At this frequency, no considerable artifacts due to discretization occur. It can be seen from Fig. 4(a) that the amplitude of the synthesized sound field does indeed decrease along the propagation direction. It can also be seen that the scattered sound field is qualitatively similar to the three-dimensional scenario depicted in Fig. 2(b).

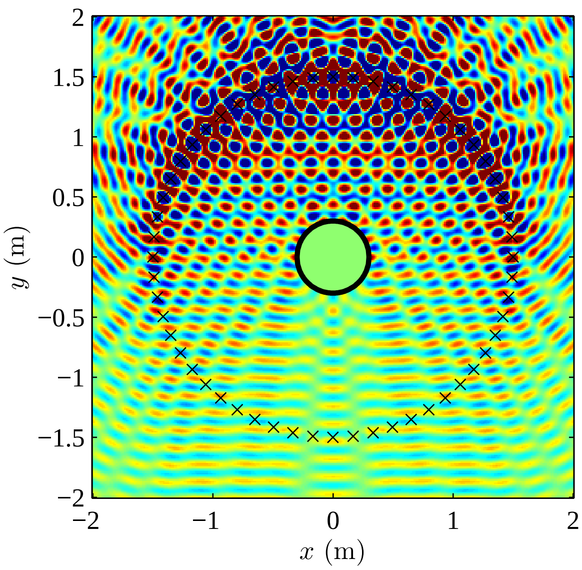
Though, the sound field seems to be *shadowed* to a stronger extent in the $2^1/_2$-dimensional scenario in Fig. 4(a) than in the three-dimensional scenario in Fig. 2(b). The presence of several listeners in a $2^1/_2$-dimensional scenario might thus have an undesired effect.

Fig. 4(b) shows the same scenario but for a frequency of $f = 2400$ Hz where the synthesized sound field is corrupted by artifacts due to spatial discretization. Note that we chose a spatially fullband driving function in Fig. 4(b) [4].

In the scenario illustrated in Fig. 4(b), not only the desired component is scattered but also the discretization artifacts.



(a) $f = 1000$ Hz



(b) $f = 2400$ Hz

Fig. 4: Sound fields the horizontal plane synthesized by a discrete circular secondary monopole distribution at different frequencies. The marks indicate the secondary sources. $R = 1.5$ m; $A = 0.3$ m

It has been shown in [14] that the discretization artifacts can be interpreted as wave fronts which impinge on a given receiver position additionally to the desired wave front. The analysis of the scenario depicted in Fig. 4(b) is therefore difficult. A time domain simulation is not revealing either since the time interval between different wave fronts is very short and impinging and scattered wave fronts superpose and make visual inspection difficult. The analysis of interaural differences is not productive since it does not consider higher level hearing mechanisms like the precedence effect or summing localization [15].

5. CONCLUSIONS

An analysis of the scattering of synthetic sound fields has been presented. It has been shown that the underlying physical mechanisms of scattering are indeed equal to the scattering of natural sound fields. However, synthetic sound field exhibit specific properties. Since the properties of synthetic sound fields can be different to the properties of natural sound fields, also the scattered sound fields can be different. One example is the circumstance that in $2^1/_2$-dimensional synthesis, the shadowing of the synthesized sound field due to a scattering object is stronger that in three-dimensional synthesis. And finally, with synthetic sound fields not only the desired component is scattered but also the artifacts which occur additionally. The consequences of the presence of several scattering objects in such a scenario can not be deduced at this stage.

6. REFERENCES

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