Pulse Compression

Performance of Delay Estimation

For accurate delay $\hat{\tau}$ estimates waveforms $\underline{s}(t)$ with

- a large bandwidth resulting in a point spread function $\underline{\chi}(t)$ with a narrow peak and
- a high energy resulting in a high SNR γ are required. Achieving a high energy with limited transmitted power requires a long duration.

$\Rightarrow$ waveforms with a large time-bandwidth product

$\Rightarrow$ the "pulse" $\underline{s}(t)$ is "compressed" by the correlator to obtain the point spread function $\underline{\chi}(t)$

Pulse Train

- pulse train, K pulses

$$\underline{s}(t) = \sum_{k=0}^{K-1} \underline{c}_k \; \underline{s}_p(t-kT)$$

$\underline{s}_p(t-kT) \rightarrow$ basic pulse

$\underline{c}_k \rightarrow$ code

- point spread function

$$\underline{\chi}(t) = \underline{s}(t) \circledast \underline{s}(t) = \int_{-\infty}^{+\infty} \underline{s}^*(\tau)\, \underline{s}(\tau+t)\, d\tau$$

$$= \int_{-\infty}^{+\infty} \sum_{l} \underline{c}_l^* \, \underline{s}_p(\tau - lT) \sum_{m} \underline{c}_m \, \underline{s}_p(\tau + t - mT)\, d\tau$$

$$= \sum_{l} \sum_{m} \underline{c}_l^* \, \underline{c}_m \int_{-\infty}^{+\infty} \underline{s}_p^*(\tau - lT)\, \underline{s}_p(\tau + t - mT)\, d\tau$$

$$= \sum_{m} \sum_{l} \underline{c}_l^* \, \underline{c}_m \int_{-\infty}^{+\infty} \underline{s}_p^*(\tau')\, \underline{s}_p(\tau' + t - (m-l)T)\, d\tau'$$

$$= \sum_{k} \underbrace{\sum_{l} \underline{c}_l^* \, \underline{c}_{l+k}}_{\underline{R}_{cc}^E(k)} \; \underbrace{\int_{-\infty}^{+\infty} \underline{s}_p^*(\tau')\, \underline{s}_p(\tau' + t - kT)\, d\tau'}_{\underline{R}_{pp}^E(t-kT)}$$

$\underline{R}_{cc}^E(k)$: correlation sequence of the code $\underline{c}_k$

$\underline{R}_{pp}^E(t-kT)$: correlation function of the basic pulse $\underline{s}_p(t)$

$$= \sum_{k} \underline{R}_{cc}^E(k)\; \underline{R}_{pp}^E(t-kT)$$

Special Case: Non-Overlapping Pulses

- basis pulse

$$\underline{s}_p(t) = \begin{cases} s_p(t) & -T/2 < t < +T/2 \\ 0 & \text{otherwise} \end{cases}$$

- correlation function of the basis pulse

$$\underline{R}^E_{pp}(t) = \begin{cases} \underline{R}^E_{pp}(t) & -T < t < +T \\ 0 & \text{otherwise} \end{cases}$$

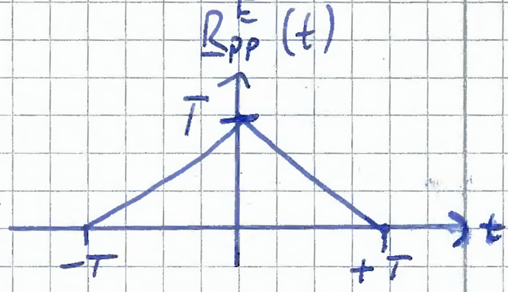
- point spread function

$$\underline{x}(kT) = \underline{R}^E_{cc}(k)\ \underline{R}^E_{pp}(0)$$

Example: Train of Non-Overlapping Rectangular Pulses

$$\underline{p}(t) = \mathrm{rect}\left(\frac{t}{T}\right)$$

$$\Rightarrow \underline{R}_{pp}^{E}(t) = T \,\Lambda\left(\frac{t}{T}\right)$$



$\Rightarrow$ The point spread function $\underline{X}(t)$ consists of straight line segments with corners at kT. The values at the corners are

$$\underline{X}(kT) = \underline{R}_{cc}^{E}(k)\; \underline{R}_{pp}(0) = T\, \underline{R}_{cc}^{E}(k).$$

$\Rightarrow$ search for codes $\underline{c}_k$ with good correlation properties

ideal code:

$$\underline{R}_{cc}^{E}(k) = \delta(k) = \begin{cases} 1 & k=0 \\ 0 & \text{otherwise} \end{cases}$$

Phase Coding

$$|\underline{c}_k| = 1 \quad , k = 0 \dots K-1$$

and

$$\underline{s}_p(t) = \mathrm{rect}\left(\frac{t}{T}\right)$$

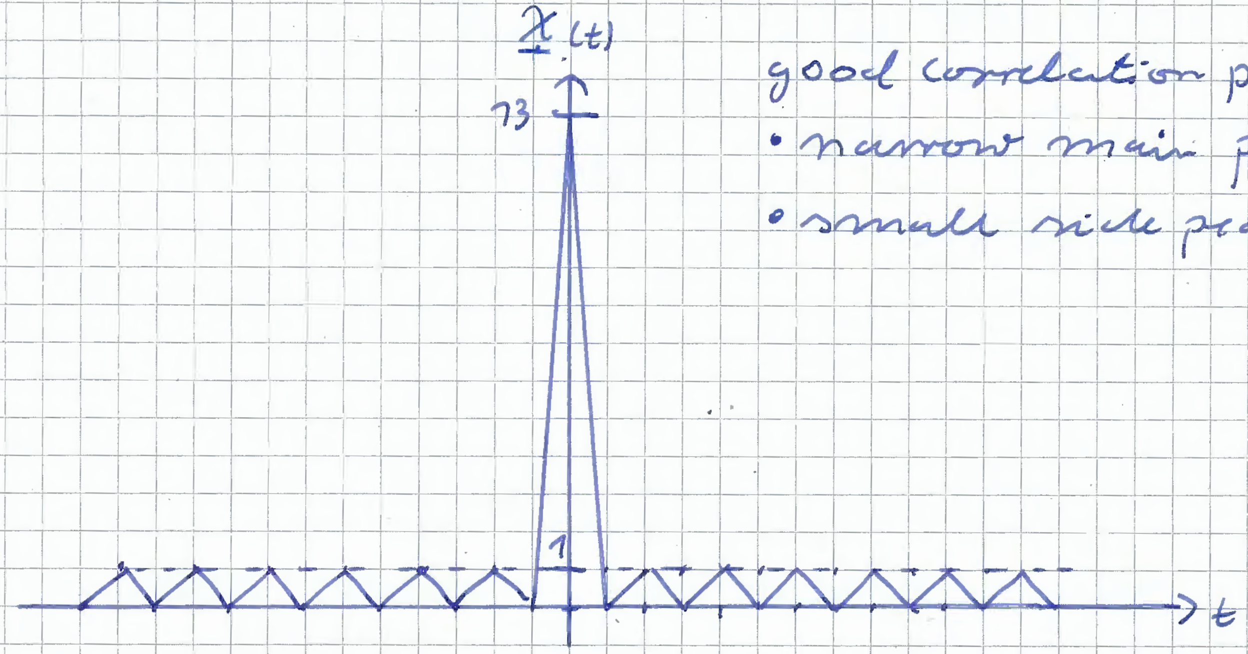
$\Rightarrow$ constant envelope $|\underline{s}(t)|$

Example: Barker codes, $\underline{c}_k \in \{-1, +1\}$

length K	code	
2	+1 −1	+1 +1
3	+1 +1 −1	
4	+1 −1 +1 +1	+1 −1 −1 −1
5	+1 +1 +1 −1 +1	
7	+1 +1 +1 −1 −1 +1 −1	
11	+1 +1 +1 −1 −1 −1 +1 −1 −1 +1 −1	
13	+1 +1 +1 +1 +1 −1 −1 +1 +1 −1 +1 −1 +1	

Example: Barker Code of Length 13

k	0	± 1	± 2	± 3	± 4	± 5	± 6	± 7	± 8	± 9	± 10	± 11	± 12
$\underline{R}_{cc}^{E}(k)$	13	0	1	0	1	0	1	0	1	0	1	0	1



good correlation properties:
- narrow main peak
- small side peaks