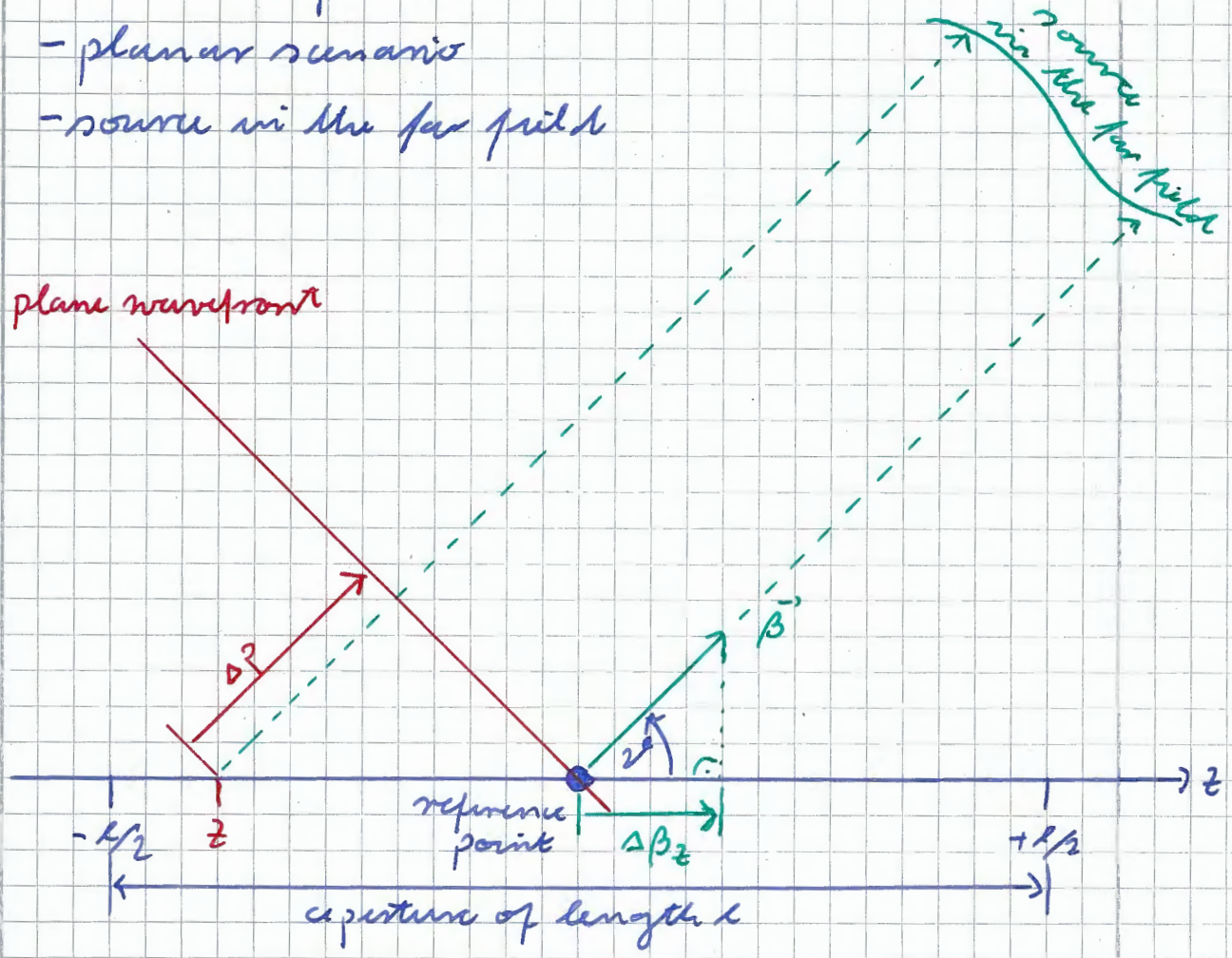
Direction of Arrival (DOA) Estimation

Aperture

for simplicity
- linear aperture
- planar scenario
- source in the far field



- wavenumber vector $\vec{\beta}$, $\|\vec{\beta}\| = \beta = \frac{2\pi}{\lambda}$
- direction of arrival (DOA) ϑ
- projection of $\vec{\beta}$ onto the aperture

$$\Delta\beta_z = \|\vec{\beta}\| \cos\vartheta = \beta \cos\vartheta \approx \beta\left(\frac{\pi}{2} - \vartheta\right)$$

($\vartheta \approx \frac{\pi}{2}$)

$\Rightarrow$ describes the direction of arrival (DOA)

- path length difference

$$\Delta p = -z \cos\vartheta$$ (in the figure z is negative)

Received Signal

- for simplicity: narrowband case
- received signal at reference point $\underline{c}_{ref}$
- time shift $\delta t = \frac{\Delta\rho}{c_0} = -\frac{z\cos\vartheta}{c_0}$
- narrowband approximation: small time shift δt corresponds to a phase shift

$$\varphi = -\omega_0 \delta t = \frac{\omega_0 z \cos\vartheta}{c_0} = \beta z \cos\vartheta = \Delta\beta_z z$$

- received signal

$$\underline{c}(z) = \underline{c}_{ref} e^{j\varphi} = \underline{c}_{ref} e^{j\cdot\Delta\beta_z z}$$

$$\underline{E}(\beta_z) = \int \underline{c}(z) e^{-j\beta_z z} dz = 2\pi\, \underline{c}_{ref}\, \delta(\beta_z - \Delta\beta_z)$$

$\Rightarrow$ spatial domain z and wavenumber domain β_z are a Fourier pair

- direction of arrival (DOA) estimation is a harmonic retrieval problem (wavenumber β_z takes the role of the frequency)
- direction of arrival (DOA) resolution

$$\approx \frac{\text{wavenumber resolution}}{\beta}$$

for $\vartheta \approx \frac{\pi}{2}$

$$\approx \frac{2\pi}{\beta \cdot \text{length of aperture}}$$

$$= \frac{\lambda}{\text{length of aperture}}$$

Summary

Mathematically
- range (delay) estimation,
- velocity (Doppler frequency) estimation, and
- direction of arrival (DOA) (wavenumber) estimation

are all harmonic retrieval problems!

⇒ the same algorithms can be used for all three problems

⇒ in the single target case, i.e., single range (delay), velocity (Doppler frequency), direction of arrival (DOA) (wavenumber) case, the incoherent correlator followed by a main peak search is optimum in the least squares sense

⇒ more advanced algorithms may be required for the multiple target case