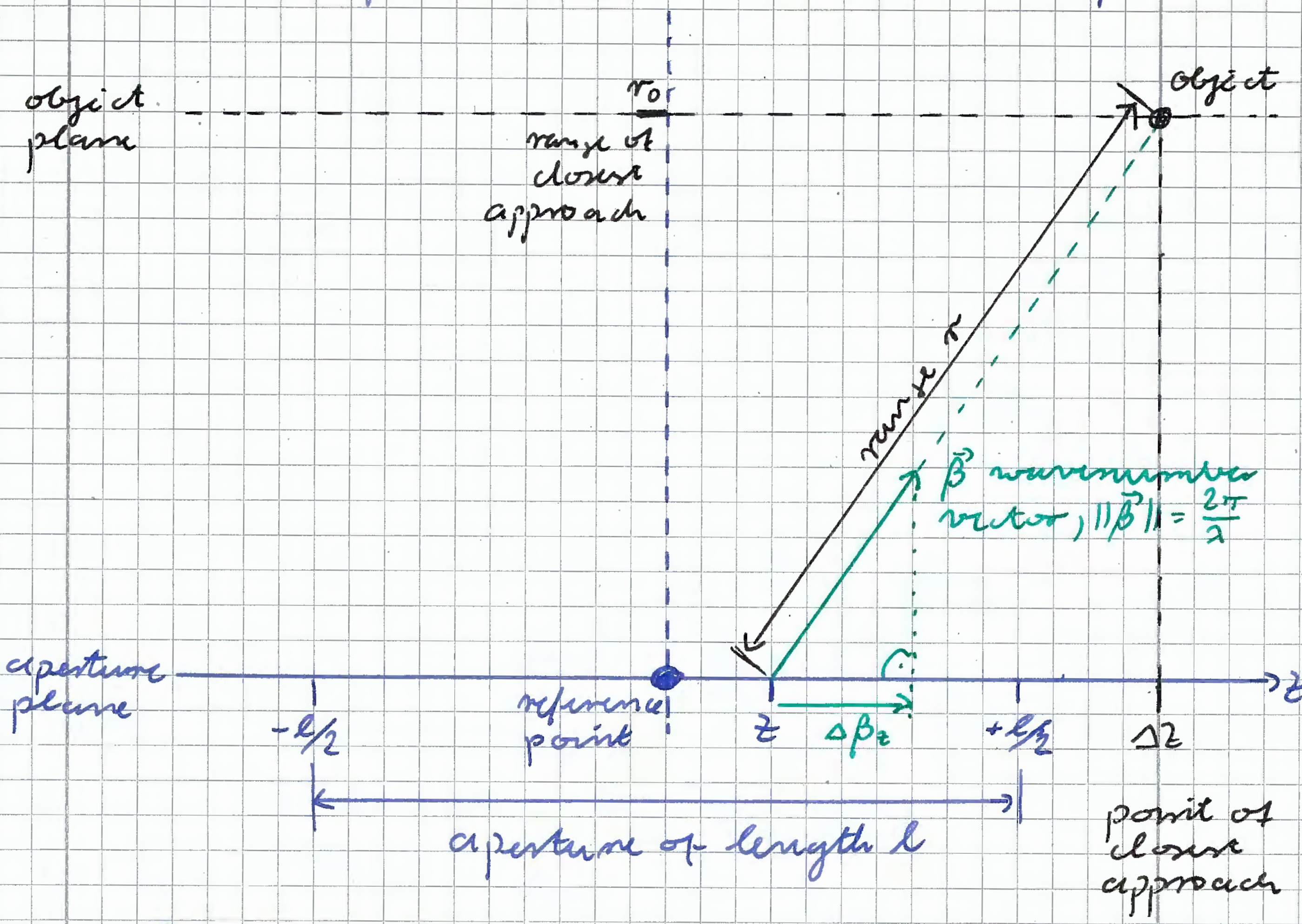
Radar Imaging

Scenario

- for simplicity one dimensional aperture



- large aperture
  - ⇒ object in the near-field
  - ⇒ wavenumber $\Delta\beta_z$ depends on z
- small aperture
  - ⇒ object in the far field
  - ⇒ wavenumber $\Delta\beta_z$ independent of z
- real aperture: fixed transmitter
  - ⇒ one way channel
- synthetic aperture: transmitter and receiver at z
  - ⇒ two way channel, additional factor 2 in the phase shifts

Range

- near-field ⇒ exact

$$r(z) = \sqrt{(z-\Delta z)^2 + r_0^2} \Rightarrow \text{hyperbola}$$

- parabolic approximation
(Fresnel approximation)

$$r(z) \approx r(z_0) + \underbrace{\frac{\partial r}{\partial z}\Big|_{z_0}}_{\frac{z_0-\Delta z}{r(z_0)}} (z-z_0) + \frac{1}{2} \underbrace{\frac{\partial^2 r}{\partial z^2}\Big|_{z_0}}_{\frac{r_0^2}{r^3(z_0)}} (z-z_0)^2$$

e.g. $z_0 = \Delta z$, $r(\Delta z) = r_0$

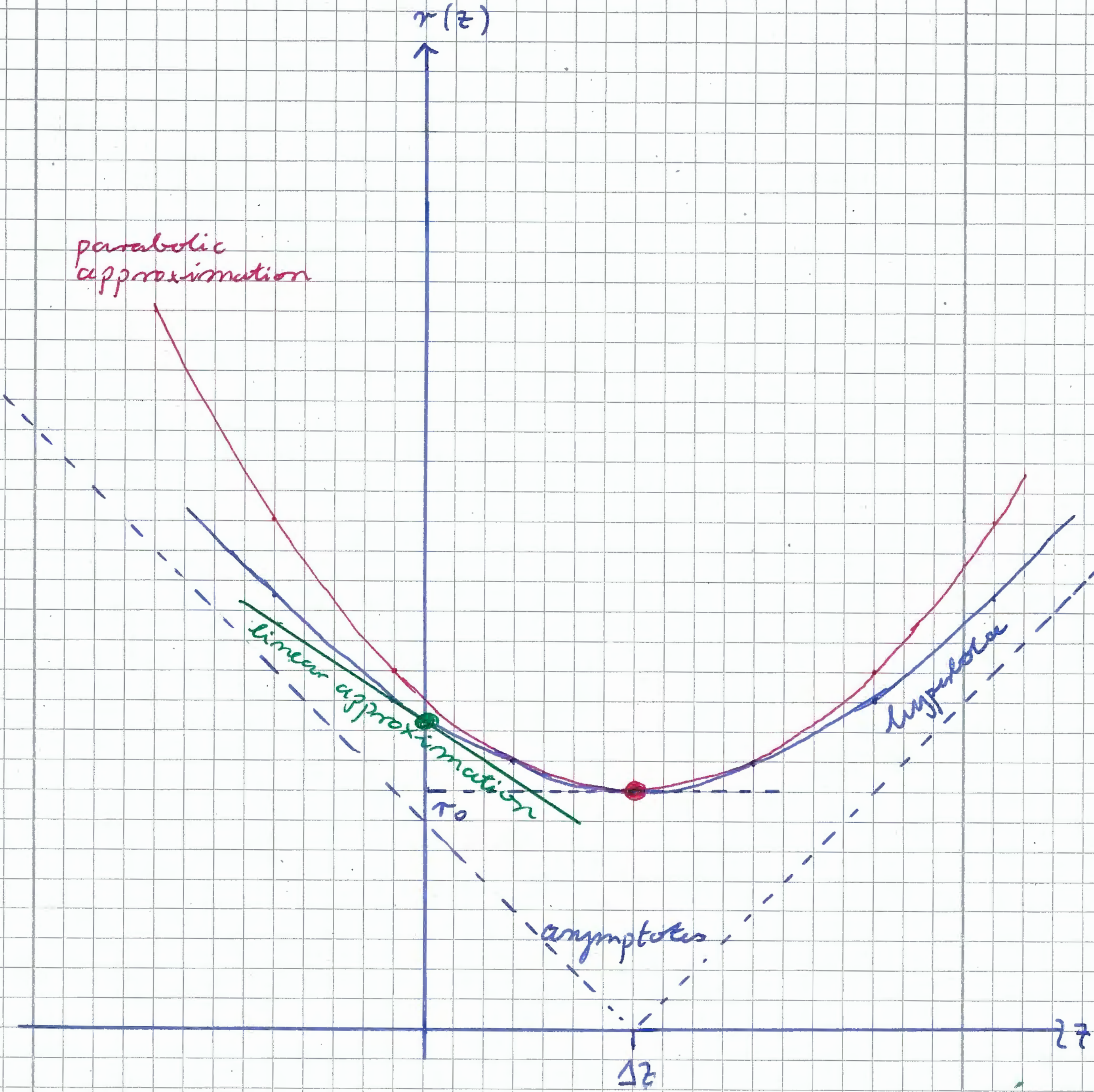
$$\Rightarrow r(z) \approx r_0 + \frac{(z-\Delta z)^2}{2r_0} = r_0 + \frac{z^2 - 2z\Delta z + \Delta z^2}{2r_0}$$

- far-field ⇒ linear approximation
(Fraunhofer approximation)

$$r(z) \approx r(z_0) + \underbrace{\frac{\partial r}{\partial z}\Big|_{z_0}}_{\frac{z_0-\Delta z}{r(z_0)}} (z-z_0)$$

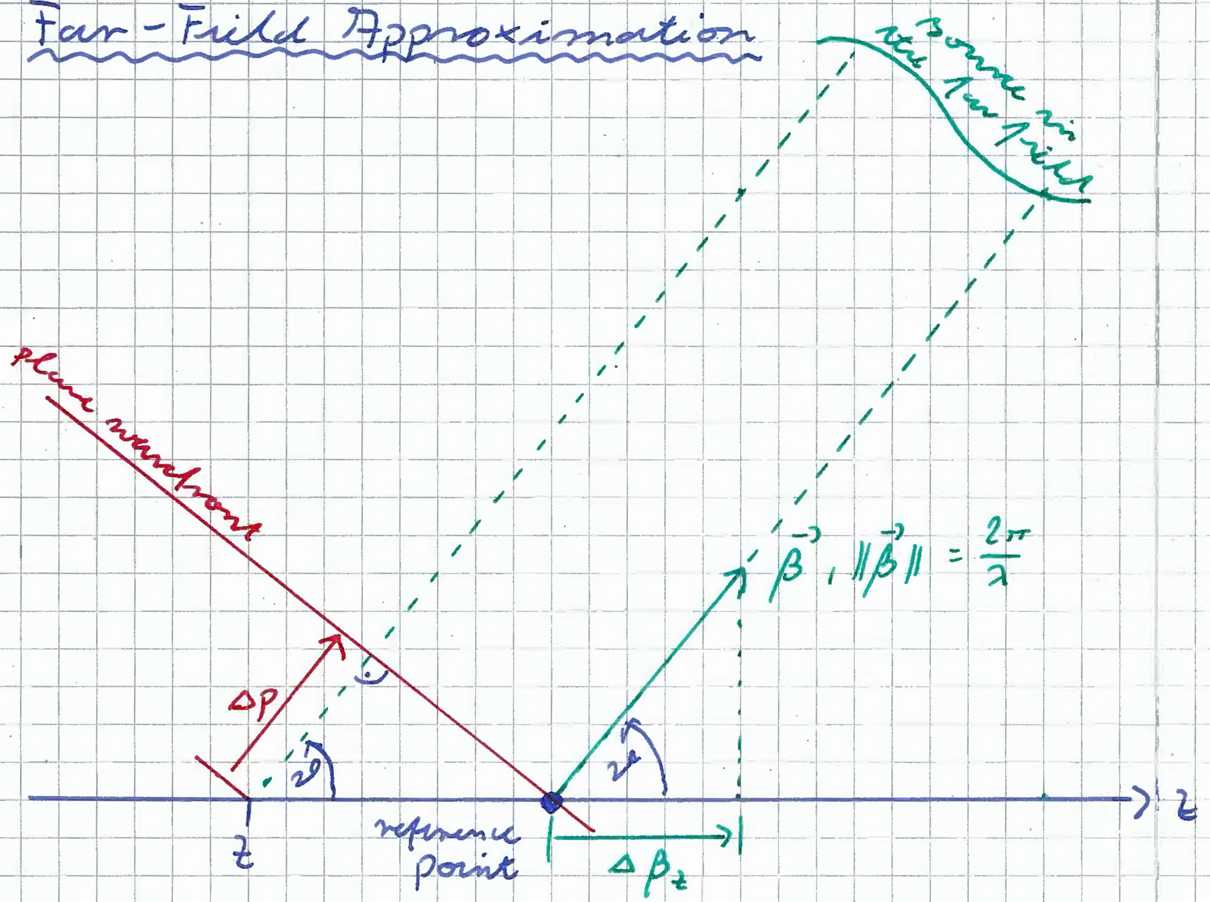
e.g. $z_0 = 0$, $r(0) = \sqrt{\Delta z^2 + r_0^2}$

$$\Rightarrow r(z) \approx r(0) - \underbrace{\frac{\Delta z}{r(0)}}_{\cos\vartheta = \frac{\Delta\beta_z}{\|\vec{\beta}\|} = \frac{\lambda}{2\pi}\Delta\beta_z} z$$



r(z)
parabolic approximation
linear approximation
hyperbola
asymptotes
r0
z
Δz

Far-Field Approximation



- one way channel
- path length difference

  $\Delta p = -z\cos\vartheta$ (in the figure z is negative)

- phase shift

  $$\varphi = -2\pi\frac{\Delta p}{\lambda} = 2\pi\frac{z\cos\vartheta}{\lambda}$$

  $$\Big|\ \text{using } \Delta\beta_z = \frac{2\pi}{\lambda}\cos\vartheta$$

  $$\varphi = \Delta\beta_z z \quad \text{linear in } z\,!$$

Received Signal, Far-Field Approximation

- one way channel
- received signal at reference point $\underline{e}_{RP}$
- narrowband approximation

$$\underline{e}(z) = \underline{e}_{RP}\, e^{-j \cdot \frac{2\pi}{\lambda}\left(r(z) - r(0)\right)}$$

- far-field approximation of $r(z)$

$$\underline{e}(z) \approx \underline{e}_{RP}\, e^{-j \cdot \frac{2\pi}{\lambda}\left(r(0) - \frac{\lambda}{2\pi}\Delta\beta_z z - r(0)\right)}$$

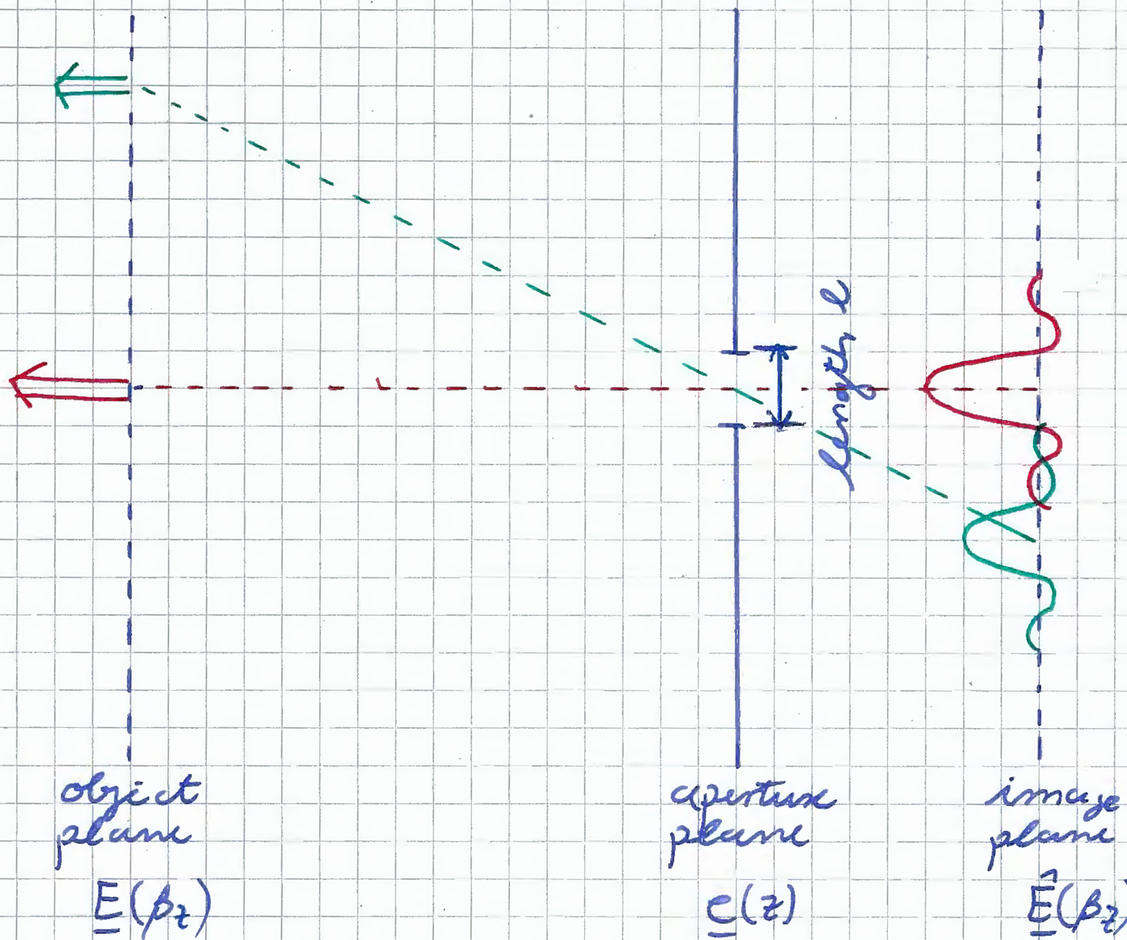
$$= \underline{e}_{RP}\, e^{j \cdot \Delta\beta_z z}$$

$$z \circ\!\!-\!\!\bullet\ \beta_z$$

$$\underline{E}(\beta_z) = \int \underline{e}(z)\, e^{-j \cdot \beta_z z}\, dz \approx 2\pi\, \underline{e}_{RP}\, \delta(\beta_z - \Delta\beta_z)$$

$\Rightarrow$ image $\underline{E}(\beta_z)$ is the Fourier transform of the signal $\underline{e}(z)$ in the aperture plane

Example: Pinhole Camera



finite aperture

- rectangular window $\mathrm{rect}\left(\frac{z}{l}\right)$ in spatial domain
- convolution with si-function in wavenumber domain!

$$\hat{\underline{E}}(\beta_z) = \mathcal{F}\left\{\underline{e}(z)\,\mathrm{rect}\left(\frac{z}{l}\right)\right\} = \mathcal{F}\{\underline{e}(z)\} * \mathcal{F}\left\{\mathrm{rect}\left(\frac{z}{l}\right)\right\}$$

$$= \underline{E}(\beta_z) * \mathrm{si}\left(\frac{\beta_z}{2}\right)$$

$\Rightarrow$ large aperture $\leftrightarrow$ high resolution (less diffraction)

as long as the far-field approximation holds

Example: Real Aperture Radar (RAR)

- In contrast to the pinhole camera, the wavefield is measured in the aperture plane using antennas.
- Often, only the brightness $|\underline{E}(\beta_z)|^2$ is of interest.
- The visibility is the autocorrelation function of the received signal $\underline{e}(z)$ in the aperture plane:

$$\mathcal{F}^{-1}\{|\underline{E}(\beta_z)|^2\} = \mathcal{F}^{-1}\{\underline{E}^*(\beta_z)\,\underline{E}(\beta_z)\}$$

$$= \underline{e}^*(-z) * \underline{e}(z) = \underline{R}^{E}_{ee}(z)$$

spatial domain		wavenumber domain
$\underline{e}(z)$	$z \circ\!\!-\!\!\bullet\ \beta_z$	$\underline{E}(\beta_z)$
$\downarrow$		$\downarrow$
$\underline{R}^{E}_{ee}(z)$	$z \circ\!\!-\!\!\bullet\ \beta_z$	$\lvert\underline{E}(\beta_z)\rvert^2$
visibility		brightness

- Typically, real apertures are relatively small
  $\Rightarrow$ far-field approximation holds but low resolution

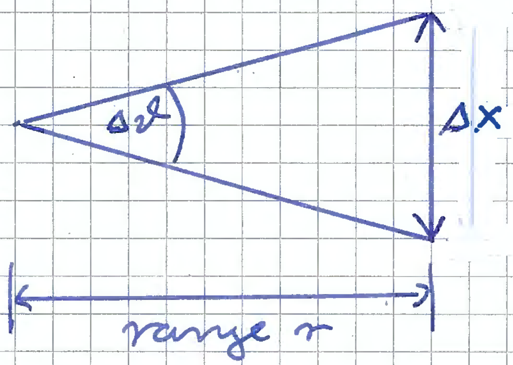
Resolution of Real Aperture Radar (RAR)

- direction of arrival (DOA) resolution

$$\Delta\vartheta \underset{\vartheta \approx \frac{\pi}{2}}{\approx} \frac{\lambda}{2\pi} \text{ wavenumber resolution}$$

$$\approx \frac{\lambda}{l}$$

- cross-range resolution



$$\Delta x \underset{\text{small } \Delta\vartheta}{\approx} r\,\Delta\vartheta \approx \frac{\lambda r}{l}$$

$\Rightarrow$ large real aperture l required

$\Rightarrow$ low resolution for large range r

$\Rightarrow$ not suitable for remote sensing

Received Signal, Parabolic Approximation

- one way channel
- parabolic approximation of $r(z)$ and $r(0)$

$$\underline{e}(z) = \underline{e}_{RP}\, e^{-j\frac{2\pi}{\lambda}\left(r(z)-r(0)\right)}$$

$$\approx \underline{e}_{RP}\, e^{-j\frac{2\pi}{\lambda}\left(r_0 + \frac{z^2-2z\Delta z+\Delta z^2}{2r_0} - r_0 - \frac{\Delta z^2}{2r_0}\right)}$$

$$= \underline{e}_{RP}\, e^{-j\frac{2\pi}{\lambda}\frac{z^2-2z\Delta z}{2r_0}}$$

- focussing (for a certain range of closest approach r_0, independent of Δz)

$$\underline{e}(z)\, \underbrace{e^{j\frac{2\pi}{\lambda}\frac{z^2}{2r_0}}}_{\substack{\approx 1 \text{ for } r_0 \to \infty \\ \text{(focussing ineffective} \\ \text{in the far-field)}}} \approx \underline{e}_{RP}\, e^{j\frac{2\pi}{\lambda}\frac{z\Delta z}{r_0}} = \underline{e}_{RP}\, e^{j\Delta\beta_z z}$$

using $\Delta\beta_z = \frac{2\pi}{\lambda}\underbrace{\frac{\Delta z}{r_0}}_{\approx \text{ const in the far field}}$

$\Rightarrow$ After focussing, the signal is the same as in the case of the far-field approximation.
The image $\underline{E}(\beta_z)$ can now be computed using the Fourier transform.

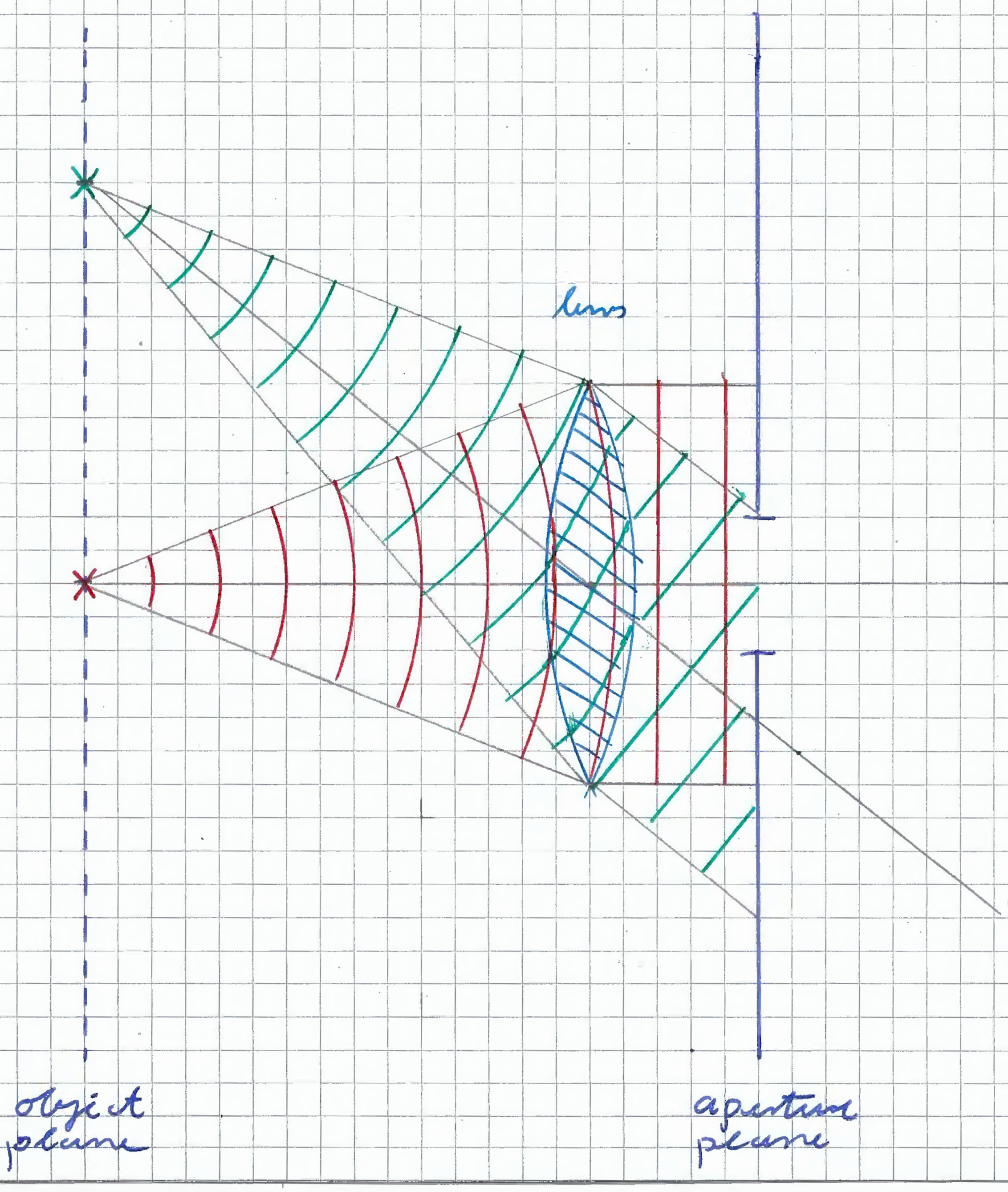
Lens

focussing introduces a

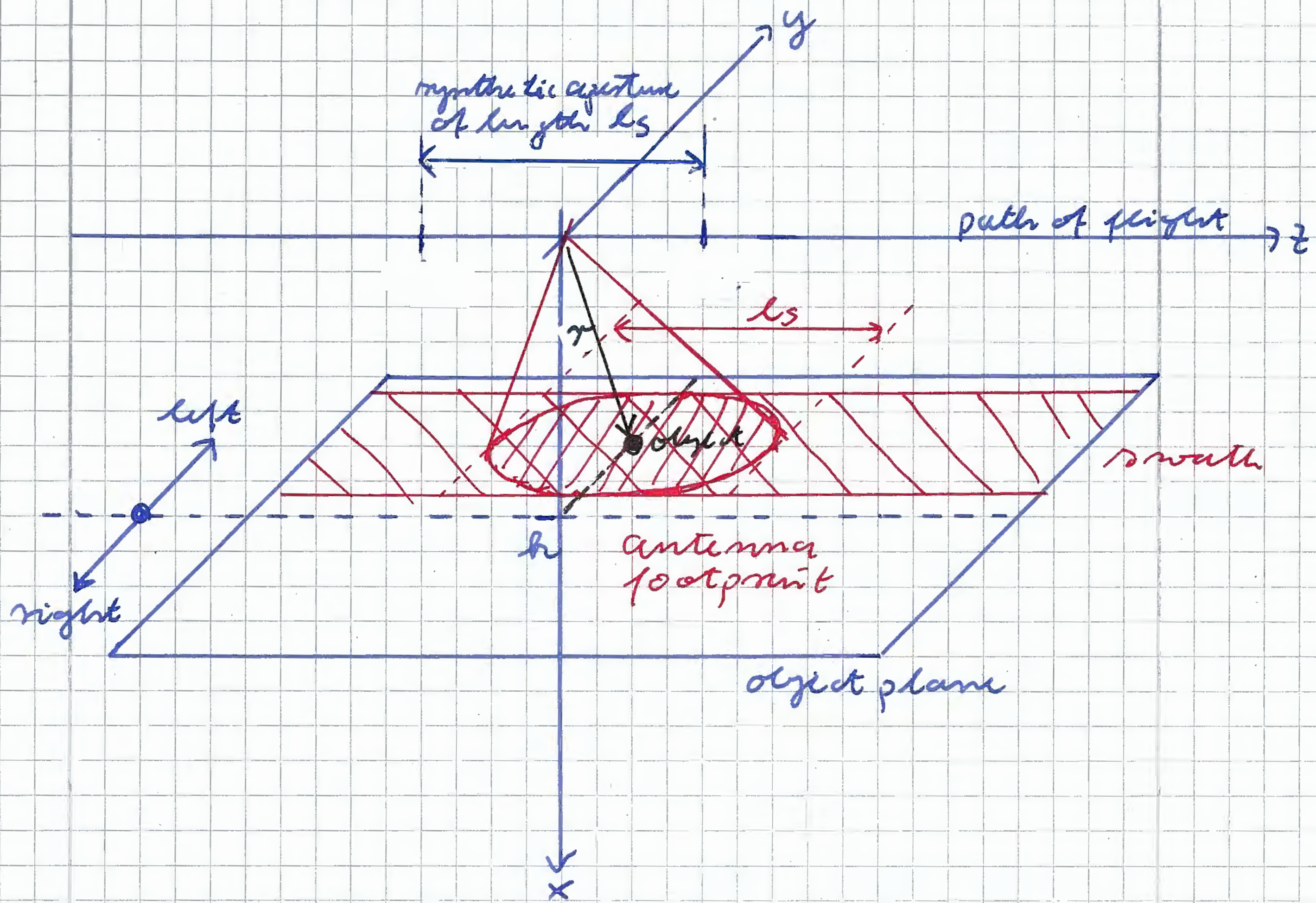
delay $\sim -z^2$

=> can be implemented using a lens

=> the spherical wave in the near-field is transformed into a plane wave (as in the far-field)



Example: Synthetic Aperture Radar (SAR)



- path of flight at height h above the object plane

- take measurements of the wavefield on the path of flight
  $\Rightarrow$ synthetic aperture

- resolve left-right-ambiguity using a directive antenna
  $\Rightarrow$ side looking synthetic aperture radar (SAR)

- length l_s of the synthetic aperture equals the length of the antenna footprint

Cone of Constant Wavenumber

The intersection of the cone of constant wavenumber $\Delta\beta_z$ and the object plane is a hyperbola of constant wavenumber $\Delta\beta_z$.

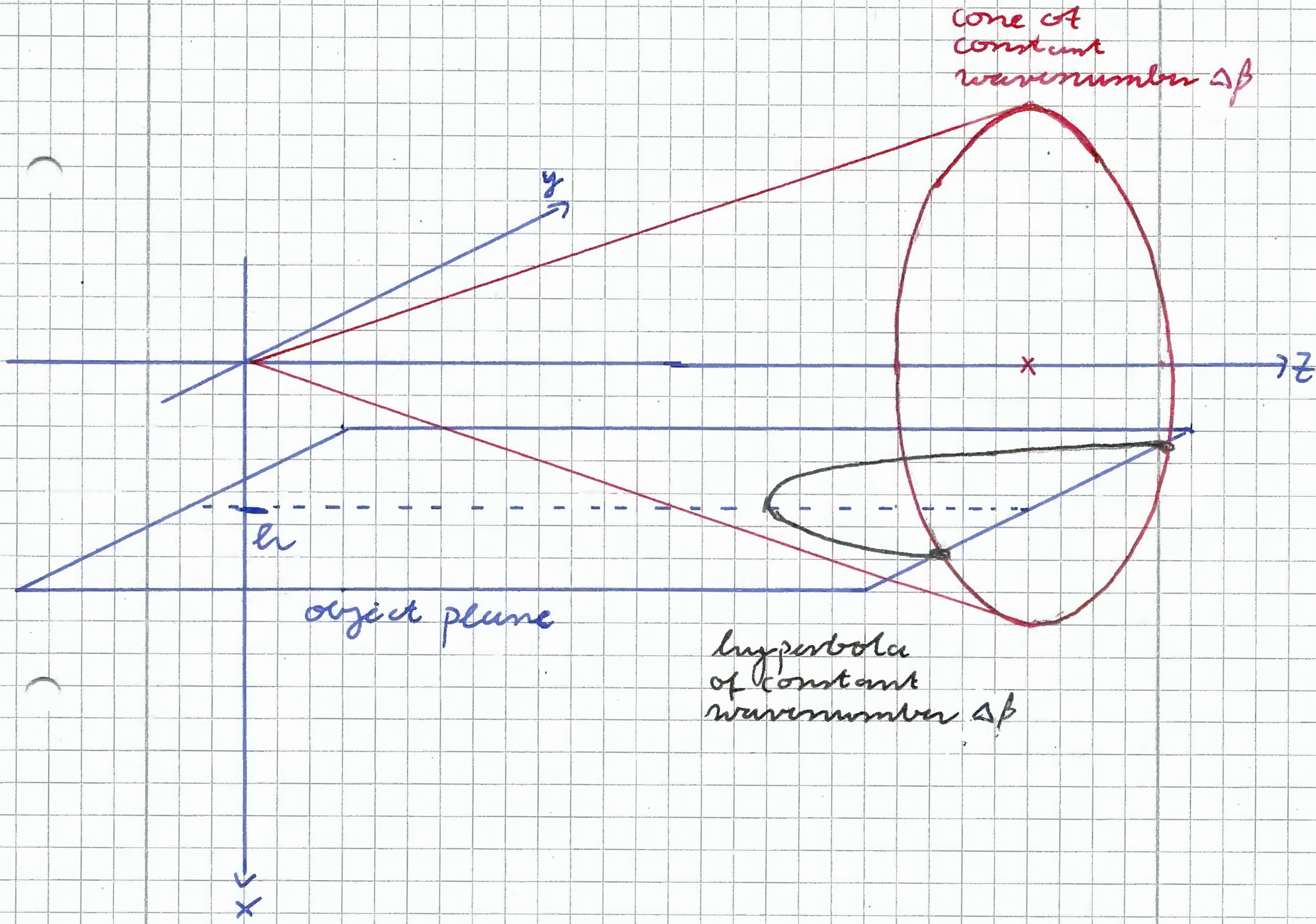


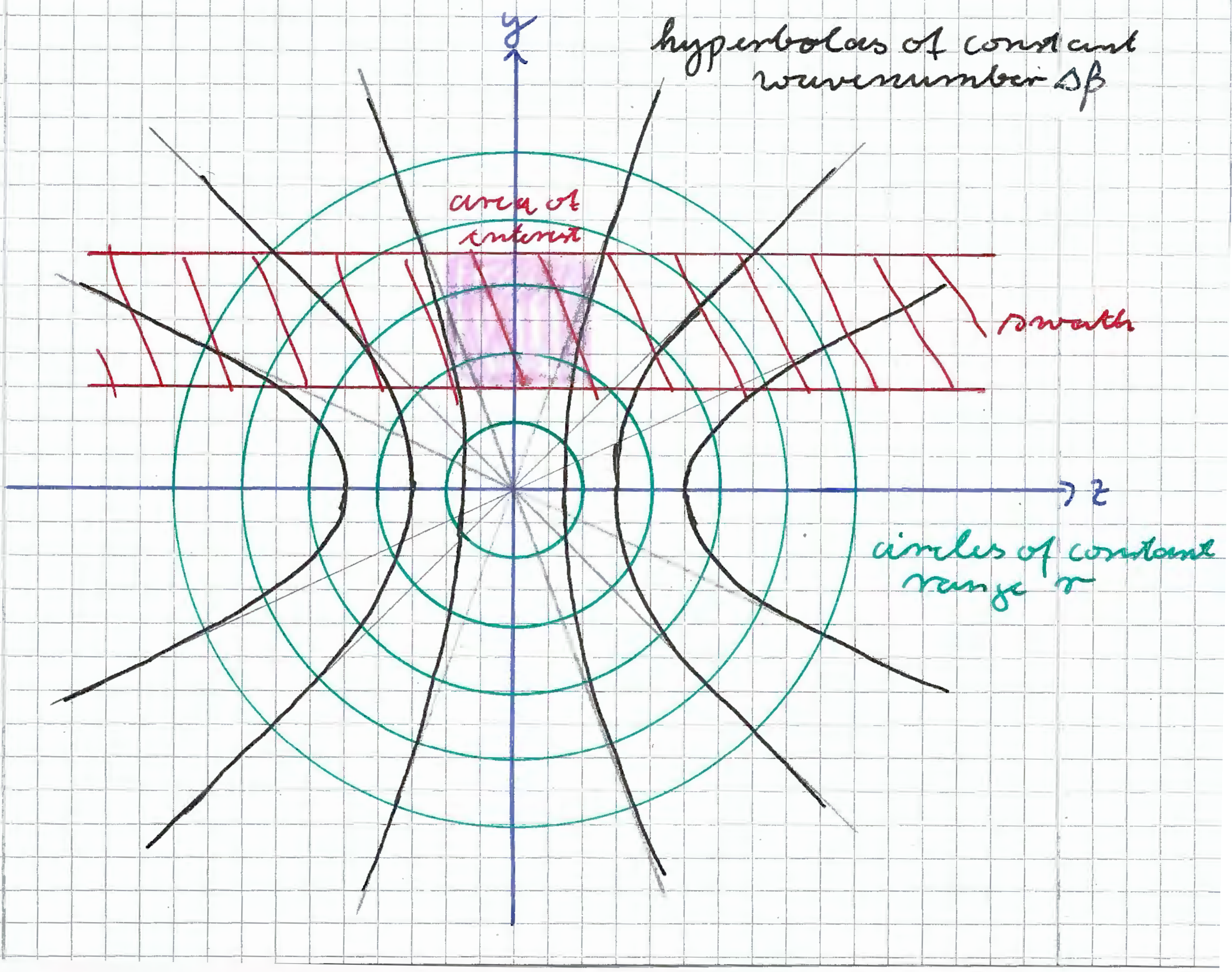
Image Formation in Synthetic Aperture Radar (SAR)

- measure
  - range r (delay Δt) and
  - wavenumber $\Delta\beta_z$ (direction of arrival ϑ, DOA)

  on the one-dimensional synthetic aperture

- compute a two-dimensional image exploiting
  - circles of constant range r and
  - hyperbolas of constant wavenumber $\Delta\beta$

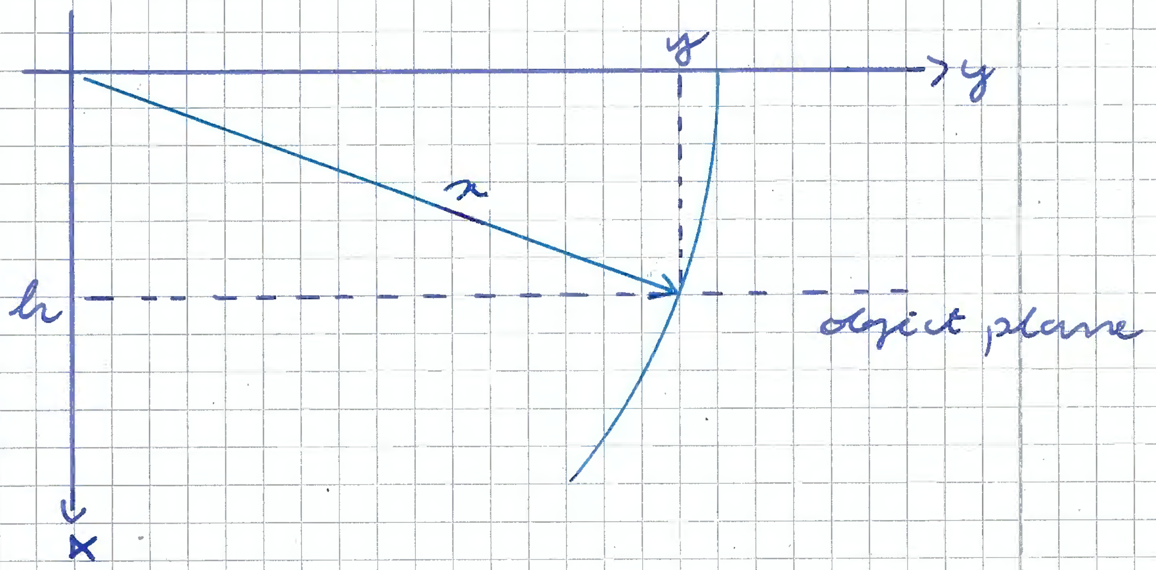
  in the object plane



Cross-Track Resolution of Synthetic Aperture Radar (SAR)

- cross-track $\hat{=}$ y
- area of interest: $\Delta z \approx 0$, $h \ll y$

$$\Rightarrow y \approx r$$



- cross-track resolution

$\approx$ range resolution

$\approx \frac{c_0}{2}$ time resolution

$$\approx \frac{\pi \, c_0}{\text{bandwidth}}$$

(factor $\frac{1}{2}$ due to two way channel)

$\Rightarrow$ independent of range r

$\Rightarrow$ suitable for remote sensing

Along-Track Resolution of Synthetic Aperture Radar (SAR)

- length l_R of real aperture
- length of the antenna footprint is equal to the length of the synthetic aperture

$$l_S = \frac{\lambda r}{l_R}$$ see cross-range resolution of real aperture radar (RAR)

- along-track $\hat{=}$ z
- area of interest: $\Delta z \approx 0$, $h \ll y$
  $\Rightarrow$ along-track $\approx$ cross-range
- along-track resolution
  $\approx$ cross-range resolution
  $$\approx \frac{\lambda r}{2 l_S} = \frac{1}{2} l_R$$
  (factor $\frac{1}{2}$ due to two way channel)
  $\Rightarrow$ smaller real apertures yield higher resolution in synthetic aperture radar (SAR)
  $\Rightarrow$ independent of range r
  $\Rightarrow$ suitable for remote sensing