Detection Theory

Binary Hypothesis Testing

Based on the observed signal y the detector has to decide whether there is a target or not.

$\Rightarrow$ two hypotheses

H_0: null hypothesis, no target present
$\Rightarrow$ only noise

H_1: alternative hypothesis, target present
$\Rightarrow$ signal plus noise

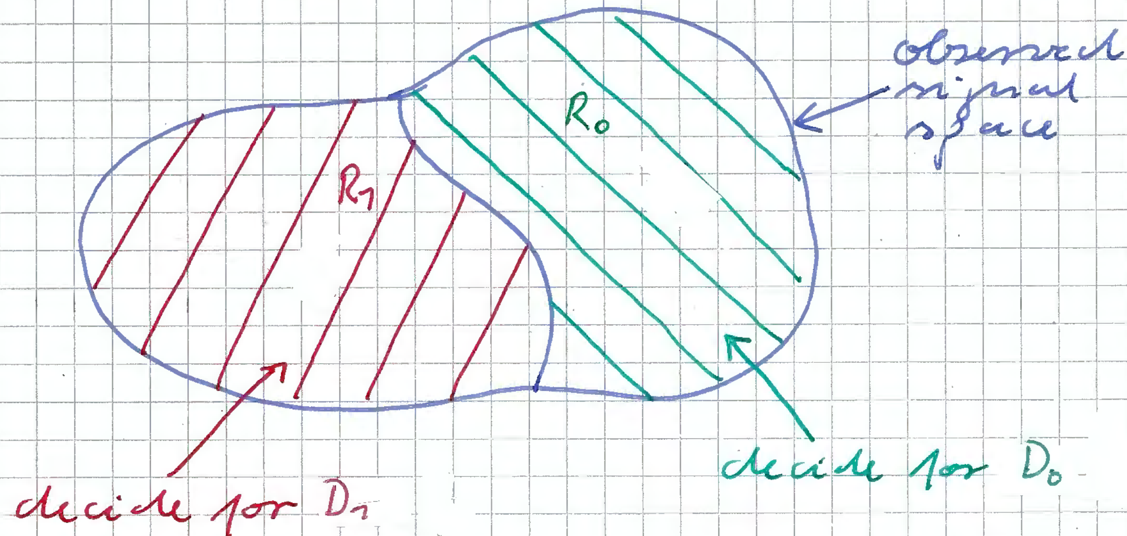
Two kinds of error with quite different consequences may occur.

type 1 error: decide for target present (D_1) even though there is no target present (H_0)
$\Rightarrow$ false positive, false alarm

type 2 error: decide for no target present (D_0) even though there is a target present (H_1)
$\Rightarrow$ false negative, missed detection

Decision Regions

- The detector is characterized by its decision regions.



- probability of false alarm

$$P_F = \Pr\{D_1 | H_0\}$$

$$= \Pr\{y \in R_1 | H_0\} = \int_{R_1} p(y|H_0)\, dy$$

- probability of missed detection

$$P_M = \Pr\{D_0 | H_1\}$$

$$= \Pr\{y \in R_0 | H_1\} = \int_{R_0} p(y|H_1)\, dy$$

- probability of detection

$$P_D = \Pr\{D_1 | H_1\}$$

$$= \Pr\{y \in R_1 | H_1\} = \int_{R_1} p(y|H_1)\, dy$$

$$= 1 - P_M$$

Design of Detectors

- find the decision regions
- A compromise inbetween minimizing both probabilities of error is required.
- idea:

$$\boxed{\text{Maximize the probability of detection } P_D \text{ for a given probability of false alarm } P_F\,!}$$

$\Rightarrow$ find the most efficient test using the method of Lagrangian multipliers

Neyman - Pearson Theorem

- maximize the Lagrange function

$$P_D - \lambda P_F = \int_{R_1} p(y|H_1)\, dy - \lambda \int_{R_1} p(y|H_0)\, dy$$

$$= \int_{R_1} \left(p(y|H_1) - \lambda\, p(y|H_0) \right) dy$$

- the decision region R_1 has to contain all observed signals y with

$$p(y|H_1) - \lambda\, p(y|H_0) > 0$$

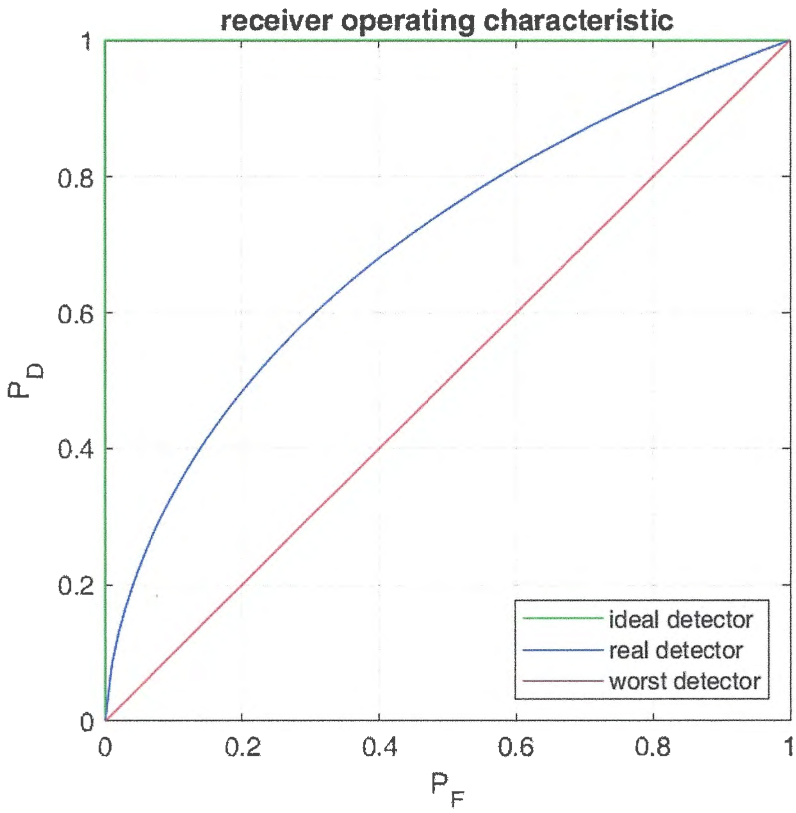
$$\Rightarrow \frac{p(y|H_1)}{p(y|H_0)} > \lambda \quad \text{likelihood ratio test}$$

- Neyman - Pearson theorem

$$\boxed{\text{The likelihood ratio test is the most efficient test.}}$$

Receiver Operating Characteristic (ROC)

Plot the obtained probability of detection P_D as a function of the given probability of false alarm P_F.



receiver operating characteristic
1
0.8
0.6
0.4
0.2
0
P_D
0
0.2
0.4
0.6
0.8
1
P_F
ideal detector
real detector
worst detector