Fluctuating Targets

Fluctuating Targets

- Typical radar targets (e.g. aircrafts) have dimensions much larger than the wavelength.
  $\Rightarrow$ They should be considered as a large number of scattering points.

- The backscattered signals interfere in a random way.
  $\Rightarrow$ The complex amplitude $\underline{A}$ of the signal at the output of the matched filter is a complex Gaussian random variable.

  $\Rightarrow$ The SNR

$$\gamma = \frac{|\underline{A}|^2}{\sigma^2}$$

is an exponentional random variable with mean $\bar{\gamma}$:

$$p_\gamma(\gamma) = \begin{cases} \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} & \gamma > 0 \\ 0 & \text{otherwise} \end{cases}$$

Decision Variable

- marginal probability density function

$$p(y \mid H_1) = \int_{-\infty}^{+\infty} \underbrace{p_{y|\gamma}(y \mid \gamma)\, p_\gamma(\gamma)}_{p_{y,\gamma}(y,\gamma)}\, d\gamma$$

- using

$$p_{y|\gamma}(y \mid \gamma) = \begin{cases} \frac{1}{\sigma^2} e^{-(\gamma + y/\sigma^2)}\, I_0\left(\frac{2\sqrt{\gamma y}}{\sigma}\right) & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

one obtains

$$p(y \mid H_1) = \begin{cases} \int_0^\infty \frac{1}{\sigma^2} e^{-(\gamma + y/\sigma^2)}\, I_0\left(\frac{2\sqrt{\gamma y}}{\sigma}\right) \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma}\, d\gamma & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{\sigma^2 \bar\gamma} e^{-y/\sigma^2} \int_0^\infty e^{-\gamma(1 + 1/\bar\gamma)}\, I_0\left(\frac{2\sqrt{\gamma y}}{\sigma}\right) d\gamma & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^\infty I_0(2\sqrt{kt})\, e^{-st}\, dt = \frac{1}{s} e^{k/s}$$

see Laplace transform of $I_0(2\sqrt{kt})$

$$= \begin{cases} \frac{1}{\sigma^2(1+\bar\gamma)} e^{-\frac{y}{\sigma^2(1+\bar\gamma)}} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

y is an exponentional random variable

Probability of Detection

$$P_D = \int_{\eta}^{\infty} p(y \mid H_1)\, dy$$

$$= \int_{\eta}^{\infty} \frac{1}{\sigma^2(1+\bar{\gamma})} e^{-\frac{y}{\sigma^2(1+\bar{\gamma})}}\, dy$$

$$= \left[- e^{-\frac{y}{\sigma^2(1+\bar{\gamma})}} \right]_{\eta}^{\infty}$$

$$= e^{-\frac{\eta}{\sigma^2(1+\bar{\gamma})}}$$

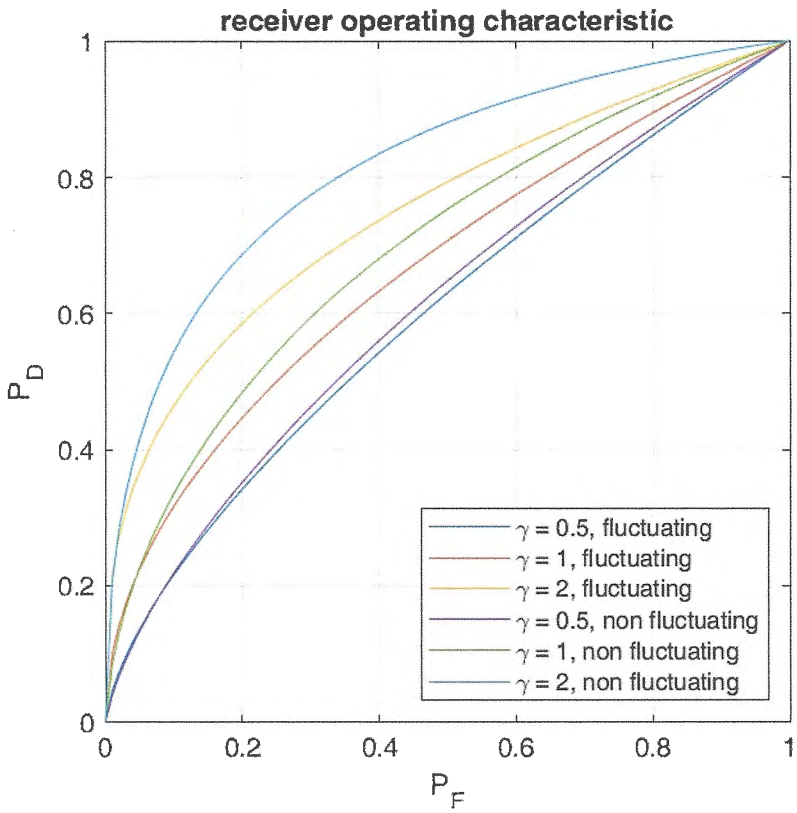
Receiver Operating Characteristic (ROC)

using

$$\eta = -\sigma^2 \ln(P_F)$$

one obtains

$$P_D = e^{\frac{\sigma^2 \ln(P_F)}{\sigma^2(1+\bar{\gamma})}} = P_F^{\frac{1}{1+\bar{\gamma}}}$$



receiver operating characteristic
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
P_D
P_F
γ = 0.5, fluctuating
γ = 1, fluctuating
γ = 2, fluctuating
γ = 0.5, non fluctuating
γ = 1, non fluctuating
γ = 2, non fluctuating