Integration

Integration

Repeat the measurements K times in order to get a more reliable detection result.

There are various methods to combine the results of the individual measurements

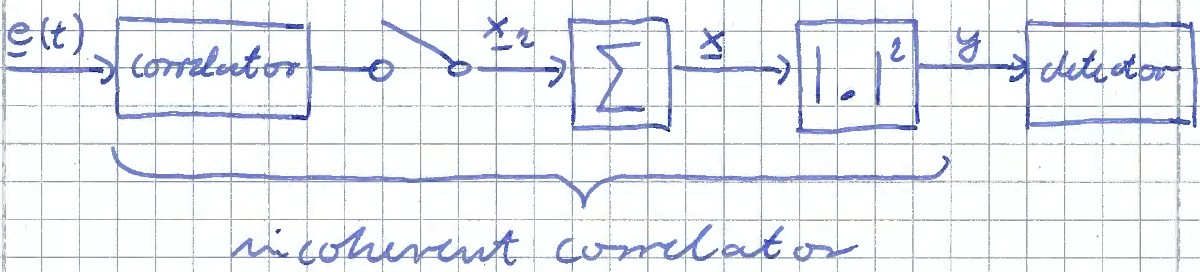
Coherent Integration

- Coherent integration can be applied if the target is constant over the whole period of measurements.

- Optimum approach: apply a correlator to the whole sequence of measurements
  $\Rightarrow$ summation of the correlator outputs $\underline{x}_k$ of the individual measurements

  $$\underline{x} = \sum_{k=1}^{K} \underline{x}_k$$

  $\Rightarrow$ SNR improvement by a factor of K (K times the energy)



- receiver operating characteristic (ROC) in the case of a non fluctuating target

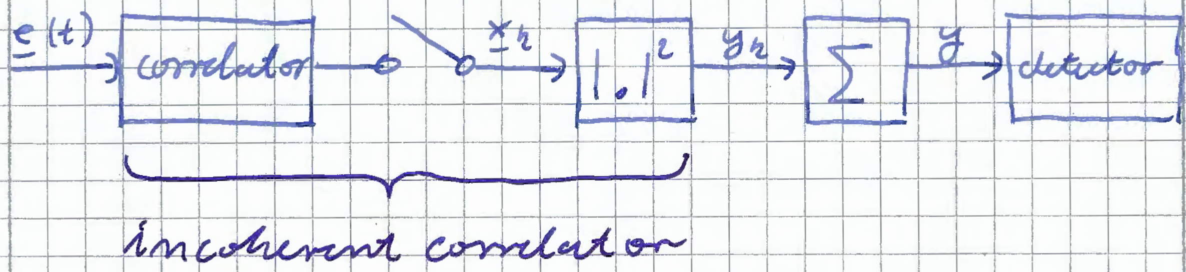
  $$P_D = Q\left(\sqrt{2K\gamma}, \sqrt{-2\ln(P_F)}\right)$$

Incoherent Integration

- Incoherent integration can also be applied to time variant targets, in particular targets whose phase is randomly changing from measurement to measurement

- Intuitive approach (suboptimum): summation of the incoherent correlator outputs y_k of the individual measurements

$$y = \sum_{k=1}^{K} y_k = \sum_{k=1}^{K} |\underline{x}_k|^2$$



Probability of False Alarm

y is a chi-square random variable with $2K$ degrees of freedom in the case of a non fluctuating target

$$P_F = \Pr\{y > \eta\} = e^{-\eta/\sigma^2} \sum_{k=0}^{K-1} \frac{1}{k!} \left(\frac{\eta}{\sigma^2}\right)^k$$

Probability of Detection

y is a noncentral chi-square random variable with 2K degrees of freedom in the case of a non fluctuating target

$$P_D = \Pr\{y > \eta\} = Q_K\left(\sqrt{2K\delta}, \sqrt{2\frac{\eta}{\sigma^2}}\right)$$

generalized Marcum's Q function

Binary Integration

Detection results are combined.

A target is detected if there are at least N hits in K measurements.

Let P_F and P_D be the probabilities of false alarm and detection, respectively, for the individual measurements.

Cumulative Probabilities

- There are $\binom{K}{N} = \frac{K!}{N!\,(K-N)!}$ possibilities to choose exactly N out of K measurements.

- cumulative probability of false alarm

$$P_{CF} = \sum_{n=N}^{K} \binom{K}{n} P_F^{\,n} \, (1-P_F)^{K-n}$$

- cumulative probability of detection

$$P_{CD} = \sum_{n=N}^{K} \binom{K}{n} P_D^{\,n} \, (1-P_D)^{K-n}$$

Special Case "N=1 out of K"

- cumulative probability of detection

$$P_{CD} = 1 - \underbrace{(1-P_D)^K}_{\text{probability of no hit}}$$

- cumulative probability of false alarm

$$P_{CF} = 1 - (1-P_F)^K$$

- linear approximation for small P_F

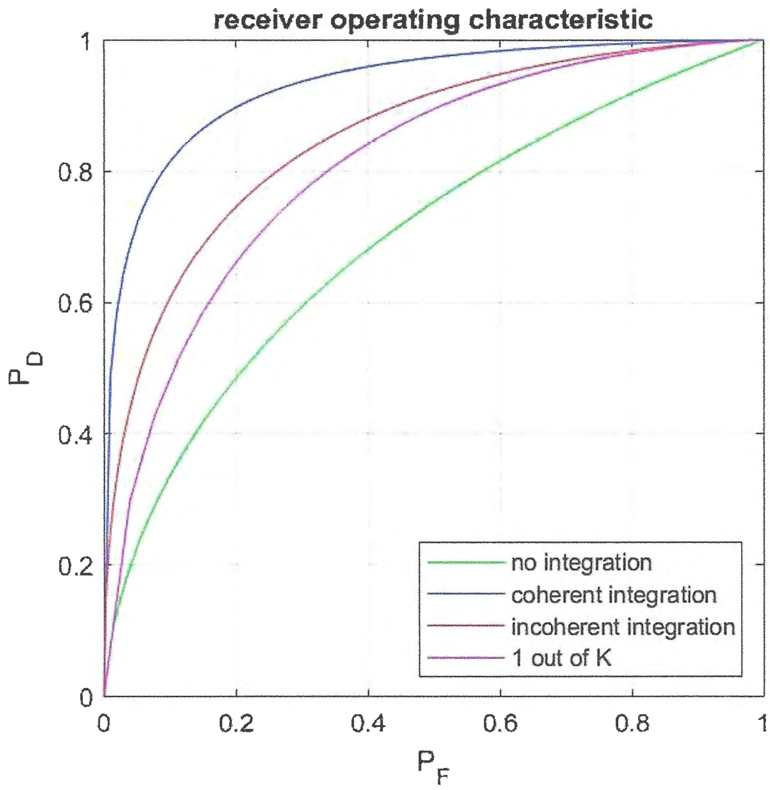
$$P_{CF} \approx K\, P_F$$

in other words: for small P_F "at least 1 out of K" is approximately equal to "exactly 1 out of K"

$$P_{CF} \approx \underbrace{\binom{K}{1}}_{=1}\; P_F^{\,1}\; \underbrace{(1-P_F)^{K-1}}_{\approx 1}$$

$$\approx K\, P_F$$

$K=4$, $\gamma=7$


receiver operating characteristic
1
0.8
0.6
0.4
0.2
0
P_D
0
0.2
0.4
0.6
0.8
1
P_F
no integration
coherent integration
incoherent integration
1 out of K