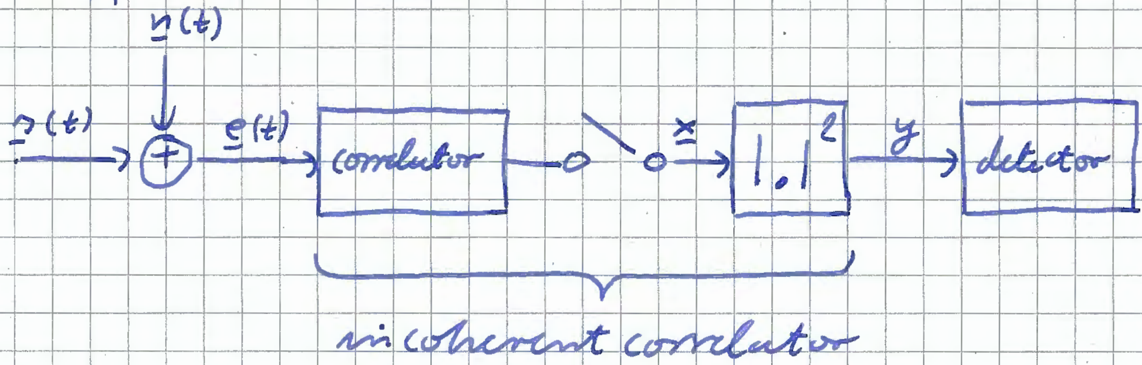
Target Detection

Incoherent Radar Receiver

- typical radar receiver



- Gaussian noise $\underline{n}(t)$

- H_0: only noise

  $\Rightarrow$ $\underline{x}$ is a complex Gaussian random variable with zero mean and variance σ^2

  $\Rightarrow$ y is an exponential random variable

$$p(y|H_0) = \begin{cases} \frac{1}{\sigma^2} e^{-y/\sigma^2} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

- H_1: signal plus noise

  $\Rightarrow$ $\underline{x}$ is a complex Gaussian random variable with mean $\underline{A}$ and variance σ^2

  $\Rightarrow$ y is a noncentral chi-square random variable with 1 degrees of freedom

$$p(y|H_1) = \begin{cases} \frac{1}{\sigma^2} e^{-\frac{|\underline{A}|^2 + y}{\sigma^2}} I_0\left(\frac{2|\underline{A}|\sqrt{y}}{\sigma^2}\right) & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

I_0: modified Bessel function of the first kind and order 0

- SNR $\quad \gamma = \frac{|\underline{A}|^2}{\sigma^2}$

$$\Rightarrow p(y|H_1) = \begin{cases} \frac{1}{\sigma^2} e^{-(\gamma + y/\sigma^2)} I_0\left(\frac{2\sqrt{\gamma y}}{\sigma}\right) & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Incoherent Detection

- likelihood ratio test

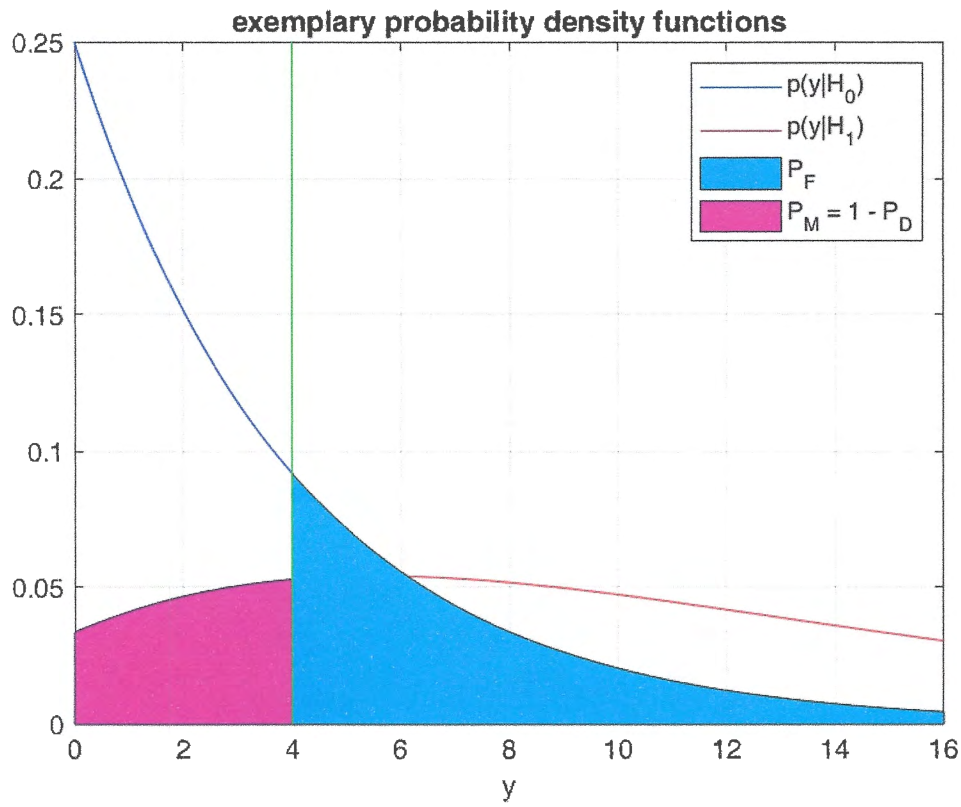
$$\frac{p(y|H_1)}{p(y|H_0)} = e^{-\gamma} I_0\left(\frac{2\sqrt{\gamma y}}{6}\right) > \lambda$$

- $I_0(.)$ is strictly growing

$\Rightarrow$ decide for H_1 if

$$y > \eta$$

with a properly chosen threshold η



$\sigma^2 = 4$

$\gamma = 2$

Probability of False Alarm

- probability of false alarm

$$P_F = \int_\eta^\infty p(y|H_0)\, dy$$

$$= \int_\eta^\infty \frac{1}{\sigma^2} e^{-y/\sigma^2} dy = \left[-e^{-y/\sigma^2}\right]_\eta^\infty$$

$$= e^{-\eta/\sigma^2}$$

- constant false alarm rate (CFAR)

$$\eta = -\sigma^2 \ln(P_F)$$

adjust the threshold η to the noise variance σ^2

Probability of Detection

- probability of detection

$$P_D = \int_{\eta}^{\infty} p(y|H_1)\, dy$$

$$= \int_{\eta}^{\infty} \frac{1}{\sigma^2} e^{-(\gamma + y/\sigma^2)} I_0\left(\frac{2\sqrt{\gamma y}}{\sigma}\right) dy$$

$$= Q\left(\sqrt{2\gamma}, \frac{\sqrt{2\eta}}{\sigma}\right)$$

Marcum's Q function

$$Q(a,b) = \int_{b}^{\infty} x\, e^{-\frac{x^2+a^2}{2}} I_0(ax)\, dx$$

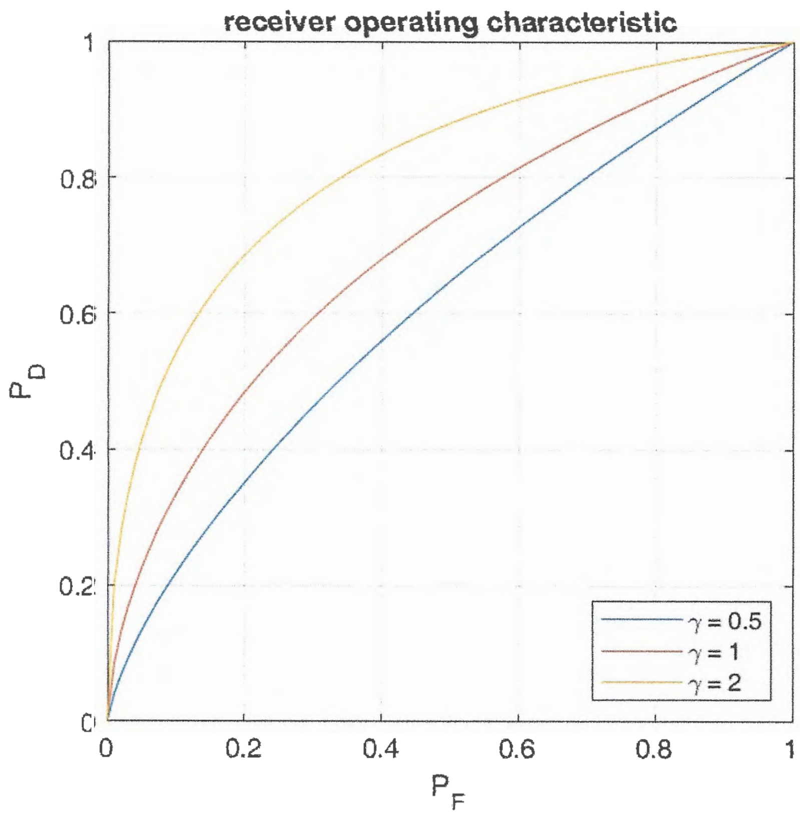
Receiver Operating Characteristic (ROC)

Using

$$\eta = -\sigma^2 \ln(P_F)$$

one obtains

$$P_D = Q\left(\sqrt{2\gamma}, \sqrt{-2\ln(P_F)}\right)$$



receiver operating characteristic
P_D
P_F
0
0.2
0.4
0.6
0.8
1
γ = 0.5
γ = 1
γ = 2