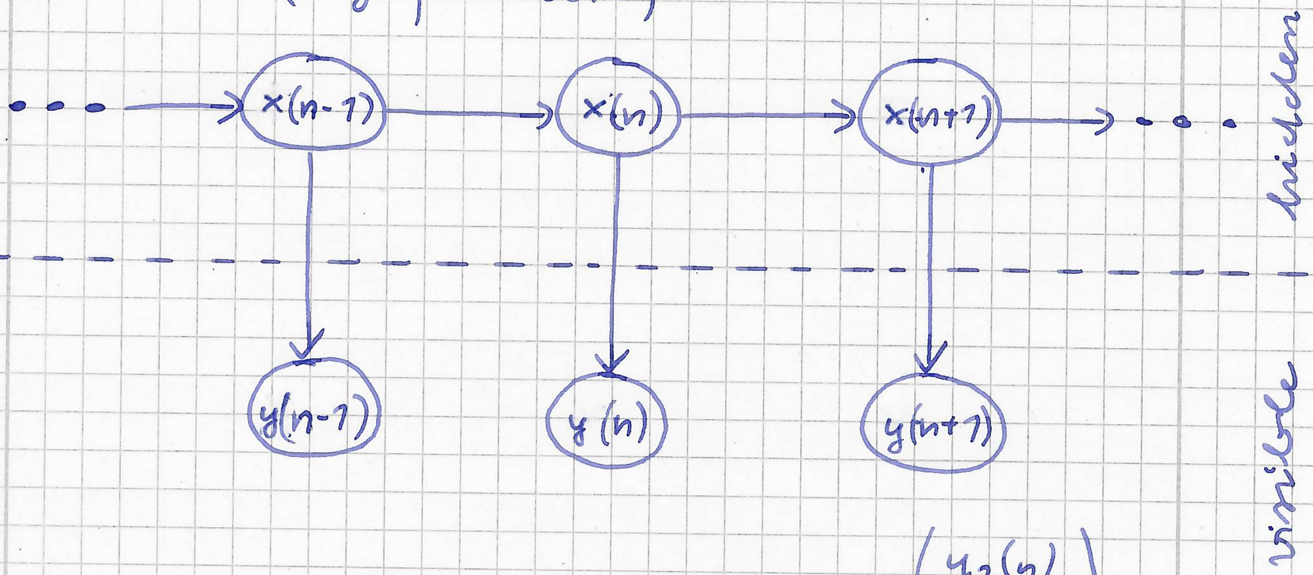
Dynamical Systems

Hidden Markov Model

$$\text{state vector } x(n) = \begin{pmatrix} x_1(n) \\ \vdots \\ x_K(n) \end{pmatrix}$$

(e.g. position)



$$\text{observation vector } y(n) = \begin{pmatrix} y_1(n) \\ \vdots \\ y_L(n) \end{pmatrix}$$

(measurements, e.g., ranges)

- discrete time index n
- arrows indicate statistical dependence

Linear Dynamical System Model

- for simplicity zero mean noise
- state transition equation

$$x(n+1) = F(n) \cdot x(n) + \underbrace{u(n)}$$

zero mean state noise, correlation matrix

$$R_{uu}(n) = E\{u(n) \cdot u^T(n)\}$$

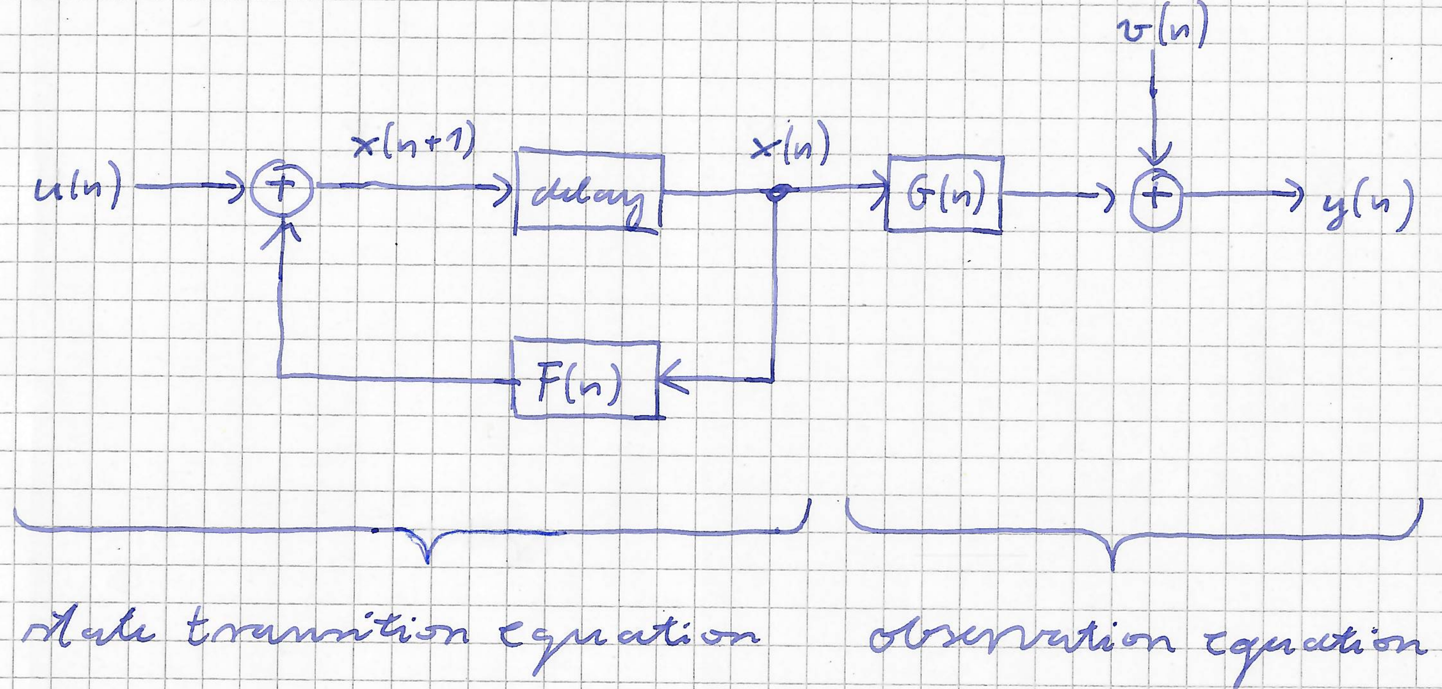
- observation equation

$$y(n) = G(n) \cdot x(n) + \underbrace{v(n)}$$

zero mean observation noise correlation matrix

$$R_{vv}(n) = E\{v(n) \cdot v^T(n)\}$$

Block Diagramm of the Linear Dynamical System Model



Kinematic Model for Target Tracking

- for simplicity 1D-scenario
- sampling interval T
- second order model:
  piecewise constant white acceleration model
- equations of motion

$$v_x(t+T) = v_x(t) + a_x(t)\,T$$

$$x(t+T) = x(t) + v_x(t)\,T + \frac{1}{2}\,a_x(t)\,T^2$$

- state transition equation

$$\underbrace{\begin{pmatrix} x(t+T) \\ v_x(t+T) \end{pmatrix}}_{x(n+1)} = \underbrace{\begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix}}_{F(n)} \cdot \underbrace{\begin{pmatrix} x(t) \\ v_x(t) \end{pmatrix}}_{x(n)} + \underbrace{\begin{pmatrix} \frac{1}{2}\,a_x(t)\,T^2 \\ a_x(t)\,T \end{pmatrix}}_{u(n)}$$

- correlation matrix of state noise

$$R_{uu}(n) = E\{u(n)\cdot u^T(n)\}$$

$$= E\{a_x^2(t)\} \begin{pmatrix} \frac{1}{4}T^4 & \frac{1}{2}T^3 \\ \frac{1}{2}T^3 & T^2 \end{pmatrix}$$