Radio Navigation and Radar

1. Exercise

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1. Problem

a) Show that

$$\mathcal{F}^{-1}(\operatorname{sign}(\omega)) = \frac{\mathrm{j}}{\pi t}$$

holds for the inverse Fourier transform of the sign-function.

The Hilbert transform of $a(t)$ is defined as

$$\hat{a}(t) = \mathcal{H}(a(t)) = \frac{1}{\pi} \int\limits_{-\infty}^{+\infty} \frac{a(\tau)}{t-\tau}\, \mathrm{d}\tau.$$

b) Show that

$$\mathcal{H}^{-1}(\hat{a}(t)) = -\mathcal{H}(\hat{a}(t))$$

holds for the inverse Hilbert transform.

c) Determine the Hilbert Transform $\hat{a}(t)$ and the analytic signal $a(t) + \mathrm{j}\hat{a}(t)$ of

$$a(t) = \operatorname{si}\left(\frac{\pi t}{T}\right).$$

d) Let $a(t)$ be a bandpass signal and $\underline{s}(t)$ be its equivalent lowpass signal. Show that the low frequency component of $a(t)\,\sqrt{2}\,\mathrm{e}^{-\mathrm{j}\omega_0 t}$ corresponds to $\underline{s}(t)$.

2. Problem

In some radar applications it is a priori known that the spectrum $\underline{S}(\omega)$ of the equivalent lowpass signal $\underline{s}(t)$ is zero for positive frequencies ω. In such situations one may save hardware complexity in the demodulator by using only a single mixer and thus only obtaining the real part $\mathrm{Re}(\underline{s}(t))$ of the equivalent lowpass signal $\underline{s}(t)$. How can one reconstruct the imaginary part $\mathrm{Im}(\underline{s}(t))$ of the equivalent lowpass signal $\underline{s}(t)$ using the Hilbert-transform in this situation?