Radio Navigation and Radar

3. Exercise

Prof. Dr.-Ing. habil. Tobias Weber

January 13, 2023
Universität Rostock

1. Problem

Let $\underline{s}(t)$ be a lowpass signal with the equivalent bandpass signal

$$a(t) = \sqrt{2}\,\mathrm{Re}\Big(\underline{s}(t)\,\mathrm{e}^{\mathrm{j}\omega_0 t}\Big)\,.$$

a) Show that $\underline{s}^*(-t)$ is the equivalent lowpass signal of the mirrored bandpass signal $a(-t)$.

Let $\underline{s}_1(t)$ and $\underline{s}_2(t)$ be two lowpass signals with the equivalent bandpass signals $a_1(t)$ and $a_2(t)$, respectively.

b) Show that the lowpass signal $\underline{s}(t)$ being equivalent to the bandpass signal

$$a(t) = a_1(t) * a_2(t)$$

can be calculated as

$$\underline{s}(t) = \frac{1}{\sqrt{2}}\underline{s}_1(t) * \underline{s}_2(t)\,.$$

The correlation function of the lowpass signal $\underline{s}(t)$ reads

$$\underline{R}^{\mathrm{E}}_{\mathrm{ss}}(t) = \int \underline{s}^*(\tau)\,\underline{s}(\tau+t)\,\mathrm{d}\tau.$$

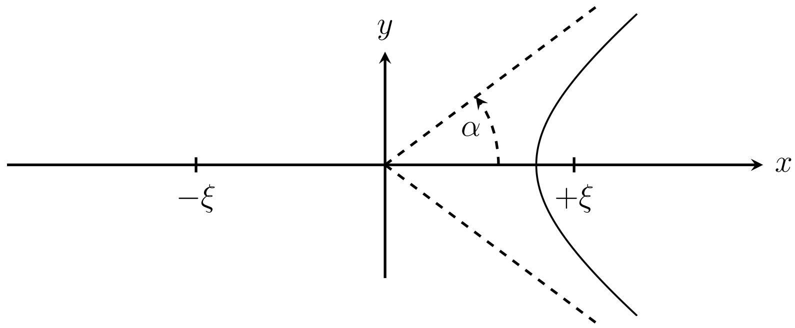
c) Determine the correlation function $R^{\mathrm{E}}_{\mathrm{aa}}(t)$ of the equivalent bandpass signal $a(t)$ as a function of the correlation function $\underline{R}^{\mathrm{E}}_{\mathrm{ss}}(t)$ and ω_0. You may exploit the fact that the correlation can be expressed using the convolution.

2. Problem

A two dimensional scenario is considered, see figure. For simplicity, the coordinates are chosen such that the two fixed points lie on the x-axis at $-\xi$ and $+\xi$. Show that the positions of constant range difference Δr lie on a hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Determine the parameters a and b of the hyperbola as functions of ξ and Δr. Determine the angle α of the asymptote as a function of ξ and Δr.



3. Problem

In the following a two dimensional bistatic radar scenario is considered, see figure. For simplicity, the coordinates are chosen such that the transmitter and the receiver lie on the x-axis at $-\xi$ and $+\xi$, respectively. Which geometrical shape do the target positions of constant total path length

$$r = r_1 + r_2$$

with r_1 being the path length from the transmitter to the target and r_2 being the path length from the target to the receiver form? Determine the equation with the parameters ξ and r which the possible target positions fulfill.