Radio Navigation and Radar

5. Exercise

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1. Problem

The ambiguity function of a signal $\underline{s}(t)$ with the energy

$$E = \int_{-\infty}^{+\infty} |\underline{s}(t)|^2 \, \mathrm{d}t$$

is defined as

$$\left|\underline{\chi}(t, \nu)\right| = \left| \int_{-\infty}^{+\infty} \underline{s}^*(\tau - t) \, \underline{s}(\tau) \, \mathrm{e}^{-\mathrm{j}\nu\tau} \, \mathrm{d}\tau \right| .$$

a) Show that

$$\left|\underline{\chi}(t, \nu)\right| \leq \left|\underline{\chi}(0, 0)\right| = E$$

holds for the maximum value.

b) Show that the symmetry relation

$$\left|\underline{\chi}(-t, -\nu)\right| = \left|\underline{\chi}(t, \nu)\right|$$

holds.

c) Show that the volume

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left|\underline{\chi}(t, \nu)\right|^2 \mathrm{d}\nu \, \mathrm{d}t = E^2$$

is constant.

d) Let the signal $\underline{s}(t)$ have the ambiguity function $\left|\underline{\chi}(t,\nu)\right|$. Show that the ambiguity function of $\underline{s}(t)\,\mathrm{e}^{\mathrm{j}\frac{1}{2}kt^2}$ can be computed as $\left|\underline{\chi}(t,\nu-kt)\right|$.

2. Problem

Compute the ambiguity functions of

a) the Gaussian pulse

$$\underline{s}(t) = \mathrm{e}^{-\frac{t^2}{4T^2}}$$

and

b) the linear frequency modulated pulse

$$\underline{s}(t) = \operatorname{rect}\left(\frac{t}{T}\right)\mathrm{e}^{\mathrm{j}\frac{1}{2}kt^2}\,.$$