Radio Navigation and Radar

1. Exercise

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1. Problem

a) Show that

$$\mathcal{F}^{-1}(\mathrm{sign}(\omega)) = \frac{\mathrm{j}}{\pi t}$$

holds for the inverse Fourier transform of the sign-function!

The Hilbert transform of $a(t)$ is defined as

$$\hat{a}(t) = \mathcal{H}(a(t)) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{a(\tau)}{t - \tau} \, \mathrm{d}\tau.$$

b) Show that

$$\mathcal{H}^{-1}(\hat{a}(t)) = -\mathcal{H}(\hat{a}(t))$$

holds for the inverse Hilbert transform!

c) Determine the Hilbert Transform of

$$a(t) = \mathrm{si}\left(\frac{\pi t}{T}\right)!$$

d) Let $a(t)$ be a bandpass signal and $\underline{s}(t)$ be its equivalent lowpass signal. Show that the low frequency component of $a(t)\sqrt{2}\,\mathrm{e}^{-\mathrm{j}\omega_0 t}$ corresponds to $\underline{s}(t)$!

2. Problem

Determine the time-bandwidth product of the triangular pulse

$$\underline{s}(t) = \Lambda(t)\,.$$