1. Problem

a) Show that
\[ \mathcal{F}^{-1}(\text{sign}(\omega)) = \frac{j}{\pi t} \]
holds for the inverse Fourier transform of the sign-function!

The Hilbert transform of \( a(t) \) is defined as
\[ \hat{a}(t) = \mathcal{H}(a(t)) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{a(\tau)}{t-\tau} \, d\tau. \]

b) Show that
\[ \mathcal{H}^{-1}(\hat{a}(t)) = -\mathcal{H}(\hat{a}(t)) \]
holds for the inverse Hilbert transform!

c) Determine the Hilbert Transform of
\[ a(t) = \text{si} \left( \frac{\pi t}{T} \right) ! \]

d) Let \( a(t) \) be a bandpass signal and \( \underline{s}(t) \) be its equivalent lowpass signal. Show that the low frequency component of \( a(t) \sqrt{2} e^{-j\omega_0 t} \) corresponds to \( \underline{s}(t) \)!
2. Problem

Determine the time-bandwidth product of the triangular pulse

\[ \mathcal{g}(t) = \Lambda(t). \]