Radio Navigation and Radar

2. Exercise

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1. Problem

Determine the autocorrelation function

$$R_{\mathrm{ww}}(t,\ \tau) = \mathrm{E}\{w(\tau)\, w(\tau + t)\}$$

of the bandpass noise

$$w(t) = \sqrt{2}x(t)\cos(\omega_0 t) - \sqrt{2}y(t)\sin(\omega_0 t)$$

as a function of ω_0 and the autocorrelation functions

$$R_{\mathrm{xx}}(t) = \mathrm{E}\{x(\tau)\, x(\tau + t)\}$$
$$R_{\mathrm{yy}}(t) = \mathrm{E}\{y(\tau)\, y(\tau + t)\}$$

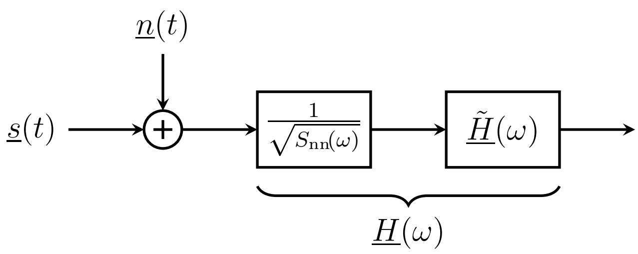
and the cross-correlation functions

$$R_{\mathrm{xy}}(t) = \mathrm{E}\{x(\tau)\, y(\tau + t)\}$$
$$R_{\mathrm{yx}}(t) = \mathrm{E}\{y(\tau)\, x(\tau + t)\}$$

of the quadrature components $x(t)$ and $y(t)$!

2. Problem

In the following a scenario is considered where the received signal results from the superposition of a known waveform $\underline{s}(t)$ and colored noise $\underline{n}(t)$ with the known power density spectrum $S_{\mathrm{nn}}(\omega)$. On the receiver side first a prewhitening filter with the transfer function $1/\sqrt{S_{\mathrm{nn}}(\omega)}$ is applied.



a) Show that the noise after the prewhitening filter is white! Determine the power density N_0 of this whitened noise.

b) Determine the transfer function $\underline{\tilde{H}}(\omega)$ of the matched filter following the prewhitening filter in such a way that the signal-to-noise ratio at the output is maximized!

c) Determine the overall transfer function $\underline{H}(\omega)$ of the receiver filter, i.e., the transfer function of the matched filter for colored noise, resulting from the concatenation of the prewhitening filter and the matched filter for the whitened noise!

3. Problem

Let $\underline{s}(t)$ be a lowpass signal with the equivalent bandpass signal

$$a(t) = \sqrt{2}\,\mathrm{Re}\Big(\underline{s}(t)\,\mathrm{e}^{\mathrm{j}\omega_0 t}\Big)\,.$$

a) Show that $\underline{s}^*(-t)$ is the equivalent lowpass signal of the mirrored bandpass signal $a(-t)$!

Let $\underline{s}_1(t)$ and $\underline{s}_2(t)$ be two lowpass signals with the equivalent bandpass signals $a_1(t)$ and $a_2(t)$, respectively.

b) Show that the lowpass signal $\underline{s}(t)$ being equivalent to the bandpass signal

$$a(t) = a_1(t) * a_2(t)$$

can be calculated as

$$\underline{s}(t) = \frac{1}{\sqrt{2}}\underline{s}_1(t) * \underline{s}_2(t)!$$

The correlation function of the lowpass signal $\underline{s}(t)$ reads

$$\underline{R}^{\mathrm{E}}_{\mathrm{ss}}(t) = \int \underline{s}^*(\tau)\,\underline{s}(\tau + t)\,\mathrm{d}\tau.$$

c) Determine the correlation function $R^{\mathrm{E}}_{\mathrm{aa}}(t)$ of the equivalent bandpass signal $a(t)$ as a function of the correlation function $\underline{R}^{\mathrm{E}}_{\mathrm{ss}}(t)$ of the lowpass signal $\underline{s}(t)$ and ω_0! You may exploit the fact that the correlation can be expressed using the convolution.

In the following a pulse train

$$\underline{s}(t) = \sum_k \underline{c}_k \underline{s}_{\mathrm{P}}(t - kT)$$

is considered. The correlation function reads

$$\underline{R}^{\mathrm{E}}_{\mathrm{ss}}(t) = \sum_k \underline{R}^{\mathrm{E}}_{\mathrm{cc}}(k)\, \underline{R}^{\mathrm{E}}_{\mathrm{PP}}(t - kT)\,,$$

where $\underline{R}^{\mathrm{E}}_{\mathrm{cc}}(k)$ is the correlation sequence of the code $\underline{c}_k$ and $\underline{R}^{\mathrm{E}}_{\mathrm{PP}}(t)$ is the correlation function of the basis pulse $\underline{s}_{\mathrm{P}}(t)$.

d) Determine the energy density spectrum $|\underline{S}(\omega)|^2$ of the lowpass signal $\underline{s}(t)$ as a function of the energy density spectrum $|\underline{S}_{\mathrm{P}}(\omega)|^2$ of the basis pulse $\underline{s}_{\mathrm{P}}(t)$ and the energy density spectrum

$$|\underline{C}(\omega)|^2 = \sum_k \underline{R}_{\mathrm{cc}}(k)\, \mathrm{e}^{-\mathrm{j}\omega kT}$$

of the code $\underline{c}_k$!