

Radio Navigation and Radar

5. Exercise

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1. Problem

The ambiguity function of a signal $\underline{s}(t)$ with the energy

$$E = \int_{-\infty}^{+\infty} |\underline{s}(t)|^2 dt$$

is defined as

$$|\underline{\chi}(\Delta t, \Delta\omega)| = \left| \int_{-\infty}^{+\infty} \underline{s}^*(t) e^{-j\Delta\omega t} \underline{s}(t + \Delta t) dt \right|.$$

a) Show that

$$|\underline{\chi}(\Delta t, \Delta\omega)| \leq |\underline{\chi}(0, 0)| = E$$

holds for the maximum value!

b) Show that the symmetry relation

$$|\underline{\chi}(-\Delta t, -\Delta\omega)| = |\underline{\chi}(\Delta t, \Delta\omega)|$$

holds!

c) Show that the volume

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |\underline{\chi}(\Delta t, \Delta\omega)|^2 d\Delta t d\Delta\omega = E^2$$

is constant!

- d) Let the signal $\underline{s}(t)$ have the ambiguity function $|\underline{\chi}(\Delta t, \Delta\omega)|$. Show that the ambiguity function of $\underline{s}(t) e^{j\frac{1}{2}kt^2}$ can be computed as $|\underline{\chi}(\Delta t, \Delta\omega - k\Delta t)|$!

2. Problem

Compute the ambiguity functions of

- a) the Gaussian pulse

$$\underline{s}(t) = e^{-\frac{t^2}{4T^2}}$$

- b) and the linearly frequency modulated pulse

$$\underline{s}(t) = \text{rect}\left(\frac{t}{T}\right) e^{j\frac{1}{2}kt^2}.$$