1. Problem

Let $x$ be a complex Gaussian random variable with nonzero mean $A$ and a variance $\sigma^2$, i.e., both real- and imaginary parts have the variance $\sigma^2/2$. The probability density function of $x$ reads

$$p_x(x) = \frac{1}{\pi \sigma^2} e^{-\frac{|x-A|^2}{\sigma^2}}.$$ 

In the following some related random variables and their probability density functions shall be studied.

a) The magnitude $x$ and the argument $\varphi$ are two functions of the two random variables real part $\text{Re}(x)$ and imaginary part $\text{Im}(x)$ of $x$. Determine the joint probability density function $p_{x,\varphi}(x, \varphi)$!

b) Show that the magnitude $x = |x|$ is a Ricean random variable with the probability density function

$$p_x(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2+|A|^2}{\sigma^2}} I_0\left(\frac{2x|A|}{\sigma^2}\right)!$$

c) Determine the probability density function $p_y(y)$ of $y = |x|^2 = x^2!$
d) Give an expression for the cumulative distribution function $\Pr\{y < \eta\}$ using Macum’s $Q$ function

$$Q(a, b) = \int_{b}^{\infty} x e^{-\frac{x^2 + a^2}{2}} I_0(ax) \, dx!$$

2. Problem

Let $x$ be a complex Gaussian random variable with zero mean and a variance $\sigma^2$, i.e., both real- and imaginary parts have the variance $\sigma^2/2$. The probability density function of $x$ reads

$$p_x(x) = \frac{1}{\pi \sigma^2} e^{-\frac{|x|^2}{\sigma^2}}.$$ 

In the following some related random variables and their probability density functions shall be studied.

a) Show that the magnitude

$$x = |x|$$

is a Rayleigh random variable with the probability density function

$$p_x(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}.$$ 

b) Determine the mean $E\{x\}$ and the variance $\text{var}\{x\}$ of $x$!

c) Determine the probability density function $p_y(y)$ of

$$y = |x|^2 = x^2.$$ 

d) Determine the mean $E\{y\}$ and the variance $\text{var}\{y\}$ of $y$!