Radio Navigation and Radar

6. Exercise

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1. Problem

Let $\underline{x}$ be a complex Gaussian random variable with nonzero mean $\underline{A}$ and a variance σ^2, i.e., both real- and imaginary parts have the variance $\sigma^2/2$. The probability density function of $\underline{x}$ reads

$$\mathrm{p}_{\underline{\mathrm{x}}}(\underline{x}) = \frac{1}{\pi\sigma^2}\,\mathrm{e}^{-\frac{|\underline{x}-\underline{A}|^2}{\sigma^2}}\,.$$

In the following some related random variables and their probability density functions shall be studied.

a) The magnitude x and the argument φ are two functions of the two random variables real part $\mathrm{Re}(\underline{x})$ and imaginary part $\mathrm{Im}(\underline{x})$ of $\underline{x}$. Determine the joint probability density function $\mathrm{p}_{\mathrm{x},\varphi}(x,\varphi)$!

b) Show that the magnitude

$$x = |\underline{x}|$$

is a Ricean random variable with the probability density function

$$\mathrm{p}_{\mathrm{x}}(x) = \frac{2x}{\sigma^2}\,\mathrm{e}^{-\frac{x^2+|\underline{A}|^2}{\sigma^2}}\,\mathrm{I}_0\!\left(\frac{2x\,|\underline{A}|}{\sigma^2}\right)!$$

c) Determine the probability density function $\mathrm{p}_{\mathrm{y}}(y)$ of

$$y = |x|^2 = x^2!$$

d) Give an expression for the cumulative distribution function $\Pr\{y < \eta\}$ using Macum's Q function

$$\mathrm{Q}(a,b) = \int_b^\infty x\,\mathrm{e}^{-\frac{x^2+a^2}{2}}\,\mathrm{I}_0(ax)\,\mathrm{d}x!$$

2. Problem

Let $\underline{x}$ be a complex Gaussian random variable with zero mean and a variance σ^2, i.e., both real- and imaginary parts have the variance $\sigma^2/2$. The probability density function of $\underline{x}$ reads

$$\mathrm{p}_{\underline{\mathrm{x}}}(\underline{x}) = \frac{1}{\pi\sigma^2}\,\mathrm{e}^{-\frac{|\underline{x}|^2}{\sigma^2}}\,.$$

In the following some related random variables and their probability density functions shall be studied.

a) Show that the magnitude

$$x = |\underline{x}|$$

is a Rayleigh random variable with the probability density function

$$\mathrm{p}_{\mathrm{x}}(x) = \frac{2x}{\sigma^2}\,\mathrm{e}^{-\frac{x^2}{\sigma^2}}!$$

b) Determine the mean $\mathrm{E}\{x\}$ and the variance $\mathrm{var}\{x\}$ of x!

c) Determine the probability density function $\mathrm{p}_{\mathrm{y}}(y)$ of

$$y = |\underline{x}|^2 = x^2!$$

d) Determine the mean $\mathrm{E}\{y\}$ and the variance $\mathrm{var}\{y\}$ of y!