Radio Navigation and Radar

7. Exercise

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1. Problem

a) Show that correlation matrices

\[ R_{XX} = E\{X \cdot X^T\} \]

are always symmetric!

b) Show that correlation matrices are always positive semidefinite!

2. Problem

A linear dynamical system model with the state transition equation

\[ X(n + 1) = F(n) \cdot X(n) + U(n) \]

and the observation equation

\[ Y(n) = G(n) \cdot X(n) + V(n) \]

is considered. Let

\[ R_{VV}(n) = E\{V(n) \cdot V(n)^T\} \]

be the correlation matrix of the observation noise \( V(n) \) and

\[ R_{XX}(n|n-1) = E\left\{ (\hat{X}(n|n-1) - X(n)) \cdot (\hat{X}(n|n-1) - X(n))^T \right\} \]

be the correlation matrix of the prediction error \( \hat{X}(n|n-1) - X(n) \).
a) The estimate is computed as
\[
\hat{X}(n|n) = \hat{X}(n|n - 1) + K(n) \cdot (Y(n) - G(n) \cdot \hat{X}(n|n - 1)).
\]
Determine \(K(n)\) in such a way that the mean square error \(E\{\|\hat{X}(n|n) - X(n)\|^2\}\) is minimized!

b) Show that the correlation matrix
\[
R_{XX}(n|n) = E\left\{ (\hat{X}(n|n) - X(n)) \cdot (\hat{X}(n|n) - X(n))^T \right\}
\]
of the estimation error \(\hat{X}(n|n) - X(n)\) can be computed as
\[
R_{XX}(n|n) = (I - K(n) \cdot G(n)) \cdot R_{XX}(n|n - 1) \cdot (I - K(n) \cdot G(n))^T
+ K(n) \cdot R_{VV}(n) \cdot K^T(n)
\]
This numerically more stable form is known as Joseph’s form.