Radio Navigation and Radar

7. Exercise

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1. Problem

a) Show that correlation matrices

$$\mathbf{R}_{\mathrm{XX}} = \mathrm{E}\Big\{\mathbf{X} \cdot \mathbf{X}^{\mathrm{T}}\Big\}$$

are always symmetric!

b) Show that correlation matrices are always positive semidefinite!

2. Problem

A linear dynamical system model with the state transition equation

$$\mathbf{X}(n+1) = \mathbf{F}(n) \cdot \mathbf{X}(n) + \mathbf{U}(n)$$

and the observation equation

$$\mathbf{Y}(n) = \mathbf{G}(n) \cdot \mathbf{X}(n) + \mathbf{V}(n)$$

is considered. Let

$$\mathbf{R}_{\mathrm{VV}}(n) = \mathrm{E}\Big\{\mathbf{V}(n) \cdot \mathbf{V}(n)^{\mathrm{T}}\Big\}$$

be the correlation matrix of the observation noise $\mathbf{V}(n)$ and

$$\mathbf{R}_{\mathrm{XX}}(n|n-1) = \mathrm{E}\Big\{\Big(\hat{\mathbf{X}}(n|n-1) - \mathbf{X}(n)\Big) \cdot \Big(\hat{\mathbf{X}}(n|n-1) - \mathbf{X}(n)\Big)^{\mathrm{T}}\Big\}$$

be the correlation matrix of the prediction error $\hat{\mathbf{X}}(n|n-1) - \mathbf{X}(n)$.

a) The estimate is computed as

$$\hat{\mathbf{X}}(n|n) = \hat{\mathbf{X}}(n|n-1) + \mathbf{K}(n) \cdot \Big(\mathbf{Y}(n) - \mathbf{G}(n) \cdot \hat{\mathbf{X}}(n|n-1)\Big)\,.$$

Determine $\mathbf{K}(n)$ in such a way that the mean square error $\mathrm{E}\Big\{\Big\|\hat{\mathbf{X}}(n|n) - \mathbf{X}(n)\Big\|^2\Big\}$ is minimized!

b) Show that the correlation matrix

$$\mathbf{R}_{\mathrm{XX}}(n|n) = \mathrm{E}\Big\{\Big(\hat{\mathbf{X}}(n|n) - \mathbf{X}(n)\Big) \cdot \Big(\hat{\mathbf{X}}(n|n) - \mathbf{X}(n)\Big)^{\mathrm{T}}\Big\}$$

of the estimation error $\hat{\mathbf{X}}(n|n) - \mathbf{X}(n)$ can be computed as

$$\begin{aligned}\mathbf{R}_{\mathrm{XX}}(n|n) = & \,(\mathbf{I} - \mathbf{K}(n) \cdot \mathbf{G}(n)) \cdot \mathbf{R}_{\mathrm{XX}}(n|n-1) \cdot (\mathbf{I} - \mathbf{K}(n) \cdot \mathbf{G}(n))^{\mathrm{T}} \\ & + \mathbf{K}(n) \cdot \mathbf{R}_{\mathrm{VV}}(n) \cdot \mathbf{K}^{\mathrm{T}}(n)!\end{aligned}$$

This numerically more stable form is known as Joseph's form.