Continuous Wave (CW) Radar
Continuous Wave (CW) Radar

- a signal is transmitted continuously

- typical block diagram - low cost

"Fletcher Processing"

\[ \text{oscillator} \rightarrow \text{amplifier} \rightarrow \text{mixer} \rightarrow \text{loosened filter} \rightarrow \text{amplifier} \]

- due to non-stable and limited dynamic range
  - only signals with a frequency being different from the instantaneous transmitted frequency can be detected
  - only strong signals from nearby scatterers can be detected

- typical applications
  - industrial level measurement
  - automotive radar
Unmodulated Continuous Wave (CW) Radar

- A sinusoidal signal of constant frequency is transmitted.
- Doppler frequencies of the received signal can be observed.
- Many targets can be detected and their velocities can be measured.

\[ \text{received signal} \]
\[ \text{transmitted signal} \]
\[ \omega_0 \quad \Delta \omega \]
\[ t \]

Attention: The frequency shift \( \Delta \omega \) may also be negative!

- Velocity:
\[ v = -\frac{\Delta \omega}{2\beta} \]

- Accuracy:

Velocity resolution \( \approx \frac{1}{2\beta} \times \frac{2\pi}{\text{measurement duration}} \)
\[ = \frac{a}{2} \times \frac{2\pi}{\text{measurement duration}} \]

(factor \( \frac{1}{2} \) due to two-way channel)

\( \Rightarrow \) depends on measurement duration and wavelength \( \lambda \)
Frequency Modulated Continuous Wave (FMCW) Radar

- measuring range requires broadband signals
  \( \Rightarrow \) frequency modulation

- linear frequency modulation
  \[
  x(t) = \mathcal{R} A \cos \left( \omega_0 t + \frac{\nu t^2}{2} \right) \\
  y(t) = A e^{j \frac{\nu t^2}{2}}
  \]

- instantaneous frequency
  \[
  \omega(t) = \frac{\partial y(t)}{\partial t} = \omega_0 + \nu t
  \]

- positive rate \( \nu = |\nu| \Rightarrow \) up-chirp
  negative rate \( \nu = -|\nu| \Rightarrow \) down-chirp
Static Scenario

- down chirp

\[ \Delta t \]
\[ \Delta \omega \]

\[ \text{duration} \]

- bandwidth = \( |R| \) duration
- frequency shift
  \[ \Delta \omega = \frac{|R|}{2} \text{st} \]
- range
  \[ r = \frac{c_0 \Delta \omega}{2 |R|} \]
- accuracy
  \[ \text{frequency resolution} \propto \frac{2\pi}{\text{duration}} \]
  \[ \text{range resolution} \propto \frac{c_0 \pi}{|R|} \frac{\text{duration}}{\text{duration}} = \frac{\pi c_0}{\text{bandwidth}} \]
  (factor \( \frac{7}{2} \) due to two way channel)
Measurement of Range and Velocity

Range-velocity ambiguity can be resolved by using a down-chirp and an up-chirp.
Linear System of Equations

down-chirp \[ \Delta u_1 = \frac{2|k|}{c_0} u - 2\beta v \]
up-chirp \[ \Delta u_2 = -\frac{2|k|}{c_0} u - 2\beta v \]

\[ \Delta u_1 + \Delta u_2 = -4\beta v \Rightarrow v = -\frac{\Delta u_1 + \Delta u_2}{4\beta} \]
\[ \Delta u_1 - \Delta u_2 = \frac{4|k|}{c_0} u \Rightarrow v = \frac{c_0}{4|k|} (\Delta u_1 - \Delta u_2) \]
Pulse Radar
Pulse Radar

- transmit a coherent train of short bandpass pulses

\[ \text{pulse repetition interval } T \]

\[ \text{pulse duration } T_p \]

\[ \text{delay } \Delta t \]

\[ \text{received signal} \]

\[ \text{transmitted signal} \]

- pulse repetition frequency

\[ \omega_R = \frac{2 \pi}{T} \]

- pulse duration

\[ T_p \ll T, \Delta t \]

- no simultaneous transmission and reception

\[ \Rightarrow \text{high performance} \]

- high instantaneous transmit power required

\[ \Rightarrow \text{high cost} \]
・Range

- Minimum delay

\[ \Delta t > T_p \]

\[ \Rightarrow \text{Minimum range} \]

\[ R_{\text{min}} = \frac{c_0}{2} T_p \]

- Maximum unambiguous delay

\[ \Delta t < T \] (more precisely \[ \Delta t < T - T_p \])

\[ \Rightarrow \text{Maximum range} \]

\[ R_{\text{max}} = \frac{c_0}{2} T = \frac{c_0 \pi}{2 \omega_{12}} \]

\[ \Rightarrow \text{Pulse repetition frequency \( \omega_{12} \) must not be too high} \]
Velocity

- Doppler frequency negligible within a single short pulse

- Doppler frequency

  \[ \Delta \omega = - \frac{2}{\beta} \nu \]

causes significant phase shift

\[ \Delta \phi = \Delta \omega T = -\frac{2}{\beta} \nu T \]

from pulse to pulse

- phase can only be measured modulo \(2\pi\)

  \[ \Rightarrow \text{ unambiguous phase } \quad |\phi| < \pi \]

- unambiguous velocity

  \[ 2 \beta \nu_{\text{max}} T < \pi \]

  \[ \nu_{\text{max}} < \frac{\pi}{2 \beta T} = \frac{\omega_2}{4 \beta} = \frac{1}{4 T} \]

  \[ \Rightarrow \text{ pulse repetition frequency } \omega_2 \]
  \[ \text{must not be too low} \]

- large unambiguous range \(\nu_{\text{max}}\)

  and large unambiguous velocity \(\nu_{\text{max}}\) are contradictory objectives
Doppler Target Indication (DTI)

Often moving targets alone are of interest. Unfortunately, they are often masked by clutter (other more or less stationary objects of the natural environment, e.g., land, sea).

Stationary clutter can be suppressed by cancellers which exploit the different Doppler signature of moving targets and clutter.
**impulse response**

\[ h_1(t) = \delta(t) - \delta(t - T) \]

**periodic transfer function**

\[
H_1(\omega) = \int_{-\infty}^{+\infty} h_1(t) e^{-j\omega t} \, dt
\]

\[
= \int_{-\infty}^{+\infty} (\delta(t) - \delta(t - T)) e^{-j\omega t} \, dt
\]

\[
= 1 - e^{-j\omega T} = e^{-j\frac{\omega}{2}} \left( e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}} \right)
\]

\[
= 2 j e^{-j\frac{\omega}{2}} \sin \left( \frac{\omega T}{2} \right)
\]

**blind velocities**

\[
\Delta \omega = -2 \beta \nu = n \frac{2\pi}{T}
\]

\[
\Rightarrow \nu = -\frac{\frac{n\pi}{T}}{\beta} = -\frac{n \frac{\pi}{T}}{2T} \quad n = \ldots, -1, 0, 1, 2, \ldots
\]
Double Cancellation

- two single cancellers in series

\[ h_2(t) = h_1(t) \ast h_1(t) \]
\[ = \delta(t) - 2 \delta(t-T) + \delta(t-2T) \]

- periodic transfer function

\[ H_2(\omega) = H_1(\omega) \ast H_1(\omega) \]
\[ = 4 e^{-\omega T} \min^2 \left( \frac{\omega T}{2} \right) \]
\[ = 4 \min^2 \left( \frac{\omega T}{2} \right) \]
\[ = 2 \left( 1 - \rho(\omega T) \right) \]
\[ |H_2(\omega)| \]

\[ -2\omega_r \quad -\omega_r \quad +\omega_r \quad +2\omega_r \]

\[ \rightarrow \omega \]

\[ \Rightarrow \text{better suppression of slow clutter} \]
Ambiguity Function
Definition of the Ambiguity Function

goal: perform a convenient and useful
concept based on their
characteristic waveform \( \varphi(t) \)

- point spread function: inner product
  of a frequency shifted and delayed
  version of \( \varphi(t) \) and \( \varphi(t) \)

\[
X(st, \omega) = \langle \varphi(t-st) e^{j \omega (t-st)}, \varphi(t) \rangle
\]

\[
= \int_{-\infty}^{\infty} \varphi^*(t-st) e^{-j \omega (t-st)} \varphi(t) \, dt
\]

\[
= \int_{-\infty}^{\infty} \varphi^*(t) e^{-j \omega t} \varphi(t+st) \, dt
\]

- ambiguity function

\[
|X(st, \omega)| = \left| \int_{-\infty}^{\infty} \varphi^*(t) e^{-j \omega t} \varphi(t+st) \, dt \right|
\]

- "ideal" ambiguity function

\[
|X(st, \omega)| = \delta(st) \delta(\omega)
\]

Unfortunately, no such waveform
exist.
Frequency Domain

- Correlation
  \[ \mathcal{F}\{e(t-at) e^{j\omega(t-at)}\} \ast e(t) \]
  \[ = \mathcal{F}\{e^*(-t-at) e^{j\omega(t+at)}\} \ast e(t) \]
  \[ = \mathcal{F}\{e^*(-(t+at)) e^{j\omega(t+at)}\} \mathcal{F}\{e(t)\} \]
  \[ = S^*(\omega-a\omega) e^{j\omega at} \delta(\omega) \]

- Ambiguity Function
  \[ |\chi(st, a\omega)| = \left| \mathcal{F}^{-1}\{S^*(\omega-a\omega) e^{j\omega at} \delta(\omega)\} \right|_{\omega = \omega_0} \]
  \[ = \left| \frac{1}{2\pi} \int_{-\infty}^{\infty} S^*(\omega-a\omega) e^{j\omega at} \delta(\omega) d\omega \right| \]
Properties of the Ambiguity Function

1.) Maximum value
\[ |X(st, \omega)| \leq |X(0, 0)| = E \]

2.) Symmetry
\[ |X(-st, -\omega)| = |X(st, \omega)| \]

3.) Constant volume
\[ \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |X(st, \omega)|^2 \, dt \, d\omega = E^2 \]

4.) Effect of chirping

Let \( \hat{h}(t) \) have the ambiguity function \( |X(st, \omega)| \).

Then the ambiguity function of \( \hat{x}(t) = \frac{1}{\sqrt{2\pi}} h(t) \) reads \( |X(st, \omega - km)| \).
Discussion

Compare waveforms of constant, e.g., normalized energy.

- According to 1.) the maximum value of the ambiguity function is constant.

- According to 3.) the volume under the squared ambiguity function is constant.

2) One may optimize waveforms in such a way that the ambiguity function is squeezed along one axis, e.g., the delay axis, but then the volume must spread out along the other axis, e.g., the frequency axis.
Cuts Through the Ambiguity Function

1.) $\Delta w = 0$

$$|X(\Delta t, 0)| = \left| \int_{-\infty}^{+\infty} \hat{X}^*(t) \hat{p}(t + \Delta t) \, dt \right|$$

$$= \left| \mathcal{R}_{\infty}^{E}(\Delta t) \right|$$

$$= \left| \mathcal{F}^{-1} \left\{ \left| \xi(\omega) \right|^2 e^{j \Delta \omega \Delta t} \right\} \right|$$

$$= \left| \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| \xi(\omega) \right|^2 e^{j \Delta \omega \Delta t} \, d\omega \right|$$

2.) $\Delta t = 0$

$$|X(0, \Delta \omega)| = \left| \int_{-\infty}^{+\infty} \hat{X}^*(t) e^{-j \Delta \omega t} \hat{p}(t) \, dt \right|$$

$$= \left| \int_{-\infty}^{+\infty} \left| \hat{X}(t) \right|^2 e^{-j \Delta \omega t} \, dt \right|$$
Example: Rectangular Pulse

- **rectangular pulse**
  \[ \sigma(t) = \text{rect}\left(\frac{t}{T}\right) \]

- **ambiguity function**
  \[
  |\hat{\sigma}(st, \omega)| = \left| \int_{-\infty}^{+\infty} \text{rect}\left(\frac{t}{T}\right) e^{-j\omega t} \text{rect}\left(\frac{t+st}{T}\right) dt \right|
  \]

  \[
  = \begin{cases} 
  \left| \int_{-\infty}^{+\infty} \text{rect}\left(\frac{t}{T-|st|} - \frac{st}{2}\right) e^{-j\omega t} dt \right| & \text{if } |st| < T \\
  0 & \text{otherwise}
  \end{cases}
  \]

  Fourier transform of \( \text{rect}\left(\frac{t}{T-|st|} - \frac{st}{2}\right) \)

  \[
  = \begin{cases} 
  \left| (T-|st|) \text{ni}(\omega(T+|st|) \frac{st}{2}) e^{-j\omega \frac{st}{2}} \right| & \text{if } |st| < T \\
  0 & \text{otherwise}
  \end{cases}
  \]

  \[
  = T \Lambda\left(\frac{st}{T}\right) \left| \text{ni}(\frac{\omega(T-|st|)}{2}) \right|
  \]
normalized ambiguity function

rectangular pulse, $T = 1$
\[ e(t) = \sum_{k=-\infty}^{\infty} e_{\nu k} \delta_\nu (t - kT) \]

\[ X(\omega, \omega_0) = \int_{-\infty}^{\infty} \sum_{\nu} \sum_{m} e_{\nu m} \int_{-\infty}^{\infty} \delta_\nu (t - \nu T) e^{-j\omega t} e^{j\omega (\tau - mT)} d\tau \]

\[ = \sum_{\nu} \sum_{m} e_{\nu m} \int_{-\infty}^{\infty} \delta_\nu (t - \nu T) e^{-j\omega (t + \nu T)} e^{j\omega (\tau - mT)} d\tau \]

\[ = \sum_{\nu} \sum_{m} e_{\nu m} \int_{-\infty}^{\infty} \delta_\nu (t - \nu T) e^{-j\omega (t + \nu T)} e^{j\omega (mT - \nu T)} d\tau \]

\[ = \sum_{\nu} \sum_{m} e_{\nu m} e^{-j\omega \nu T} \int_{-\infty}^{\infty} \delta_\nu (t - \nu T) e^{j\omega (\tau - \nu T)} d\tau \]

\[ = X_\nu (\omega - \nu \cdot \omega_0) \]

- point spread function
- point spread function of the pulse
- point spread function of the pulse
Unmodulated pulses

\[ x_2 = \begin{cases} 1 & h = 0, \ldots, K-1 \\ 0 & \text{otherwise} \end{cases} \]

Point spread function

\[ x(t, \omega) = \sum_{k} \sum_{l} x_k^* \Delta(t - kT_1) e^{-j \omega \Delta t} \times \chi_{P} (at - kT_1) \]

\[ = \sum_{k=1}^{K-1} \sum_{l=0}^{K-1} x_k^* \Delta(T_1) e^{-j \omega \Delta t} \times \chi_{P} (at - kT_1) \]

\[ + \sum_{k=1}^{K-1} \sum_{l=0}^{K-1} x_k^* \Delta(T_1) e^{-j \omega \Delta t} \times \chi_{P} (at - kT_1) \]

\[ = \sum_{k=1}^{K-1} \chi_{P} (at - kT_1) \Delta(T_1) e^{-j \omega \Delta t} \frac{1}{K-1} \sum_{l=0}^{K-1} e^{-j \omega \Delta t} \]

\[ + \chi_{P} (at) \Delta(T_1) \sum_{l=0}^{K-1} e^{-j \omega \Delta t} \]

\[ + \sum_{k=1}^{K-1} \chi_{P} (at - kT_1) \Delta(T_1) \sum_{l=0}^{K-1} e^{-j \omega \Delta t} \]
**Geometric Series**

\[
\sum_{k=0}^{K-1} e^{-j \omega k T} = \frac{1 - e^{-j \omega (K-1) T}}{1 - e^{-j \omega T}}
\]

\[
e^{-j \omega \frac{K-1}{2} T} \frac{e^{j \omega \frac{K-1}{2} T} - e^{-j \omega \frac{K-1}{2} T}}{e^{j \omega T/2} - e^{-j \omega T/2}}
\]

\[
= (K-1) e^{-j \omega \frac{K-1}{2} T} \frac{\min(\omega \frac{K-1}{2} T)}{(K-1) \min(\omega T/2)}
\]

**Dirichlet Function**

\[
din(x) = \frac{\min\left(\frac{N x}{2}\right)}{N \min\left(\frac{x}{2}\right)}
\]

**Properties:**

1. \( din(x) = din(-x) \)

2. \( din(0) = \lim_{N \to \infty} \frac{\min\left(\frac{N x}{2}\right)}{N \min\left(\frac{x}{2}\right)} = \lim_{x \to 0} \frac{N \arccos\left(\frac{N x}{2}\right)}{N \arccos\left(\frac{x}{2}\right)} = 1 \)

3. \( din(x + 2\pi) = (-1)^N din(x) \)
ambiguity function

\[ |X(a_t, s\omega)| \]

\[ = \left| \sum_{n=-K+1}^{K+1} X_p(at-hT, s\omega) (K-1h1) e^{-j\cdot s\omega \frac{K-1-1h1-2hT}{2}} \right| \]

\[ + X_p(at, s\omega) K e^{-j\cdot s\omega \frac{K-1}{2} T} \]

\[ + \sum_{n=1}^{K} X_p(at-hT, s\omega) (K-1h1) e^{-j\cdot s\omega \frac{K-1-1h1}{2} T} \]

\[ = \left| \sum_{n=-K+1}^{K-1} X_p(at-hT, s\omega) (K-1h1) e^{-j\cdot s\omega \frac{K-1-hT}{2} T} \right| \]
* Non-overlapping complex ambiguity function of the broad pulse, i.e. time limited broad pulse

\[
X_p(\alpha t, \omega) = \begin{cases} 
X_p(0, \omega) & -T/2 < \alpha t < T/2 \\
0 & \text{otherwise}
\end{cases}
\]

\[
\Rightarrow |X(\alpha t, \omega)| = \sum_{k=-K}^{K-1} |X_p(\alpha t-kT, \omega)| |\delta_{k,kT}(\omega T)|
\]
Normalized ambiguity function

Unmodulated train of rectangular pulses, $K = 5, T = 1, T_p = 0.5$

Compare pulse radar ramp and velocity ambiguity
Radar Imaging
For simplicity one dimensional aperture

- Large aperture
  - Object in the near-field
  - Wavenumber $A \beta$ depends on $z$

- Small aperture
  - Object in the far-field
  - Wavenumber $A \beta$ independent of $z$

- Real aperture: fixed transmitter
  - One way channel

- Synthetic aperture: transmitter and receiver at $z$
  - Two way channel, additional factor 2 in the phase shifts

Reference point $z = 0$
Range

• near-field => exact

\[ r(z) = \sqrt{(z-z_0)^2 + r_0^2} \Rightarrow \text{hyperbola} \]

• parabolic approximation (Fresnel approximation)

\[ r(z) \approx r(z_0) + \frac{2r_0}{\theta_0} \sqrt{\frac{z_0 - z}{r(z_0)}} + \frac{2}{\theta_0^2} \left( \frac{z_0 - z}{r(z_0)} \right)^2 \]

e.g., \( z_0 = 4z \) \( r(4z) = r_0 \)

\[ \Rightarrow r(z) \approx r_0 + \frac{(z-z_0)^2}{2r_0} = r_0 + \frac{z^2 - 2 z_0 z + z_0^2}{2r_0} \]

• far-field => linear approximation (Fraunhofer approximation)

\[ r(z) \approx r(z_0) + \frac{2r_0}{\theta_0} \left( z - z_0 \right) \]

e.g., \( z_0 = 0 \) \( r(0) = \sqrt{A z^2 + r_0^2} \)

\[ \Rightarrow r(z) \approx r(0) - \frac{A z^2}{r(0)} \]

\[ \cos \theta_0 \approx \frac{\alpha r_0}{\| \beta \|} = \frac{1}{2r_0} \| \beta \| ^2 \]
Far-Field Approximation

- One way channel
- Path length difference

\[ \Delta p = -r \cos \lambda \] (in the figure \( r \) is negative)

- Phase shift

\[ \varphi = -2\pi \frac{\Delta p}{\lambda} = 2\pi \frac{r \cos \lambda}{\lambda} \]

\[ \text{unity: } \Delta \beta_x = \frac{2\pi}{\lambda} \cos \lambda \]

\[ \varphi = \Delta \beta_x \lambda \text{ linear in } \frac{r}{\lambda} \]
Received Signal, Far-Field Approximation

- one way channel
- received signal at reference point $E_{12p}$
- narrowband approximation

$$ e^2(z) = e^{2p} e^{-j \frac{2\pi}{\lambda} (r(z) - r(0))} $$

- far-field approximation of $r(z)$

$$ e^2(z) \propto e^{2p} e^{-j \frac{2\pi}{\lambda} (r(0) - \frac{\alpha}{2\pi} \sigma_\beta^2 z - r(0))} $$

$$ e^2(z) \propto e^{2p} e^{-j \sigma^2 z} $$

$$ E(\beta_z) = \int e^2(z) e^{-j \sigma \beta_z^2} d\beta_z = 2\pi \epsilon_{12p} \delta(\beta_z - \Delta \beta) $$

$\Rightarrow$ image $E(\beta_z)$ is the Fourier transform of the signal $e^2(z)$ in the aperture plane
finite aperture

- rectangular window \( \text{rect}(\frac{z}{w}) \) in spatial domain
- convolution with \( \pi \)-function in wavenumber domain:
  \[
  \hat{E}(\beta_2) = \mathcal{F}\{e(z) \text{rect}(\frac{z}{w})\} = \mathcal{F}\{e(z)\} \ast \mathcal{F}\{\text{rect}(\frac{z}{w})\} = \hat{E}(\beta_2) \ast \pi\left(\frac{\beta_2}{w}\right)
  \]

\( \Rightarrow \) large aperture \( \Rightarrow \) high resolution (less diffraction)

as long as the far-field approximation holds
Example: Real Aperture Radar (12.1.*12)

- In contrast to the parabolic reflector, the wavefield is measured in the aperture plane using antennas.
- Often, only the brightness $|E(\beta)|^2$ is of interest.
- The visibility is the autocorrelation function of the received signal $\zeta(z)$ in the aperture plane:

$$
\hat{F}^{-1}\{ |E(\beta_z)|^2 \} = \hat{F}^{-1}\{ E^*(\beta_z) E(\beta_z) \}
$$

$$
= \zeta^*(-z) * \zeta(z) = L^E \zeta(z)
$$

- Typically, real apertures are relatively small.
- Far-field approximation holds but low resolution.
Resolution of Wide Aperture Radar (WARR)

- Direction of arrival (DOA) resolution

\[ \Delta \theta \approx \frac{\lambda}{2L} \quad \text{wavenumber resolution} \]

\[ \theta \approx \frac{\pi}{2} \]

\[ \approx \frac{\lambda}{l} \]

- Cross-range resolution

\[ \Delta x \approx \frac{x}{\lambda} \approx \frac{2\pi}{k} \]

\[ \text{small \ \set} \]

\[ \Rightarrow \text{large \ \text{real aperture} \ \ell \ \text{required}} \]

\[ \Rightarrow \text{low \ \text{resolution} \ \text{for large range} \ \chi} \]

\[ \Rightarrow \text{not \ \text{suitable} \ \text{for remote sensing}} \]
Received Signal: Parabolic Approximation

- one way channel

- parabolic approximation of $r(x)$ and $r(0)$

$$e(x) = e^{j \frac{2\pi}{\lambda} (r(x) - r(0))}$$

$$\approx e^{j \frac{2\pi}{\lambda} (r_0 + \frac{x^2 - 2x x_0 + x_0^2}{2 r_0} - r_0 - \frac{a x^2}{2 r_0})}$$

$$= e^{j \frac{2\pi}{\lambda} \frac{x^2 - 2x x_0}{2 r_0}}$$

- low-curvature (for a certain range of closest approach $r_0$, independent of $a x$)

$$e^{j \frac{2\pi}{\lambda} \frac{x^2}{2 r_0}} = e^{j \frac{2\pi}{\lambda} \frac{\Delta x^2}{2 r_0}} = e^{j \Delta \beta x}$$

$$\approx 1 \text{ for } r_0 \to \infty$$

(low-curvature ineffective in the far-field)

$$\Delta \beta = \frac{2\pi}{\lambda} \frac{\Delta x}{r_0}$$

$$\propto \cos \beta \text{ in the far-field}$$

=) After low-curvature, the signal is the same as in the case of the far-field approximation.

The image $E(\beta)$ can now be computed using the Fourier transform.
To minimize introducing a delay $\sim t^2$

\[ \Rightarrow \text{can be implemented using a lens} \]

\[ \Rightarrow \text{the spherical wave in the near-field is transformed into a plane wave} \]

(as in the far-field)
- Path of flight at height \( h \) above the object plane

- Take measurements of the wavefield on the path of flight
  \( \Rightarrow \) synthetic aperture

- Resolve left-right ambiguity using a directive antenna
  \( \Rightarrow \) side looking synthetic aperture radar (SAR)

- Length \( L_s \) of the synthetic aperture equals the length of the antenna footprint
Cone of Constant Wavenumber

The intersection of the cone of constant wavenumber $\Delta \beta$ and the object plane is a hyperbola of constant wavenumber $\Delta \beta$. 

[Diagram showing a cone intersecting a plane to form a hyperbola.]
Image Formation in Synthetic Aperture Radar (SAR)

- Measure
  - range \( r \) (delay \( \sigma_t \)) and
  - wavenumber \( \beta_z \) (direction of main lobe)
  on the one-dimensional synthetic aperture

- Compute a two-dimensional image exploiting
  - circles of constant range \( r \) and
  - hyperbolas of constant wavenumber \( \beta_z \)
in the object plane

\[ \begin{align*}
\text{hyperbolas of constant wavenumber } \beta_z \\
\text{circles of constant range } r
\end{align*} \]
Cross-Track Resolution of Synthetic Aperture Radar (SAR)

- cross-track \( \approx y \)
- area of interest: \( 0 \leq x, b \ll y \)

\[ \Rightarrow y \times r \]

- cross-track resolution

\[ \approx \text{range resolution} \]

\[ \approx \frac{c_0}{2} \times \text{time resolution} \]

\[ \approx \frac{B}{c_0} \]

(bandwidth)

(factor \( \frac{1}{2} \) due to two way channel)

\( \Rightarrow \) independent of range

\( \Rightarrow \) suitable for remote sensing
Along-Track Resolution of Synthetic Aperture Radar (SAR)

- length LR of real aperture
- length of the antenna footprint is equal to the length of the synthetic aperture
  \[ l_s = \frac{2x}{LR} \]
  see cross-range resolution of real aperture radar (124.12)
- along-track ≈ \( \frac{x}{y} \)
- area of interest: \( 0 < x < y \)
- along-track ≈ cross-range
- along-track resolution
  \[ x \text{ cross-range resolution} \]
  \[ \approx \frac{2x}{2l_s} = \frac{1}{2} LR \]
  (factor \( \frac{1}{2} \) due to two-way channel)

- smaller real apertures yield higher resolution in synthetic aperture radar (SAR)
- independent of range \( r \)
- suitable for remote sensing