Correlated Sources

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- example:
  signals impinging from different directions of arrival come from the same transmitter
  - multipath propagation
  - radar (static targets)

- possible consequence:
  rank deficient source correlation matrix
  $\operatorname{rank}(\underline{R}_{RP}) < K$

- problem:
  $$R = \operatorname{rank}(\underline{R}_{ss}) = \operatorname{rank}(\underline{A} \cdot \underline{R}_{RP} \cdot \underline{A}^{*T})$$
  $$= \min(N, \operatorname{rank}(\underline{R}_{RP}))$$
  The noise free correlation matrix may have a rank $R < K$, i.e., it does not span the full signal subspace. Only certain linear combinations of the array manifold vectors can be observed.

  "signals from different sources always superimpose in a similar way"

Spatial Smoothing

- prerequisite:
  several identical subarrays

- idea:
  "signals from different sources superimpose differently at different subarrays"

- noise free correlation matrix of the m-th subarray:

$$\underline{R}_{ss}^{(m)} = \underline{A}^{(m)} \cdot \underline{R}_{RP} \cdot \underline{A}^{(m)*T}$$

$$= \underline{A}_{RP} \cdot \underline{\phi}^{(m)} \cdot \underline{R}_{RP} \cdot \underline{\phi}^{(m)*T} \cdot \underline{A}_{RP}^{*T}$$

- averaged noise free correlation matrix:

$$\underline{R}_{ss} = \frac{1}{M} \sum_m \underline{R}_{ss}^{(m)}$$

$$= \underline{A}_{RP} \cdot \underbrace{\frac{1}{M} \sum_m \underline{\phi}^{(m)} \cdot \underline{R}_{RP} \cdot \underline{\phi}^{(m)*T}}_{\text{full rank if } M \geq K} \cdot \underline{A}_{RP}^{*T}$$

full rank if $M \geq K$
$\Rightarrow \underline{R}_{ss}$ spans the signal subspace

- signal processing point of view:
  compute an average (sample) correlation matrix of the subarrays

```
function Ravg = spatialsmoothing(Ree, M)
%SPATIALSMOOTHING spatial smoothing
%   requires a uniform linear array
%   Ravg: averaged covariance matrix
%   Ree: covariance matrix
%   M: number of subarrays

N = length(Ree); % number of antenna elements

Ravg = zeros(N-M+1); % initialization

for m = 1:M
    Ravg = Ravg+Ree(m:N-M+m, m:N-M+m)/M;
end

end
```

Forward Backward Averaging

- prerequisite:
  centrosymmetric array, conjugate centrosymmetric array manifold vectors
- array manifold matrix:

$$\Pi \cdot \underline{A}^* = \underline{A}$$

- noise free backward correlation matrix:

$$\begin{aligned}\underline{R}_{ss,B} &= E\{\Pi \cdot \underline{s}^* \cdot \underline{s}^T \cdot \Pi^{*T}\} \\ &= \Pi \cdot E\{\underline{A}^{*T} \cdot \underline{s}_{RP}^{*T} \cdot \underline{s}_{RP}^T \cdot \underline{A}^T\} \cdot \Pi^{*T} \\ &= \Pi \cdot \underline{A}^{*T} \cdot \underline{R}_{RP}^* \cdot \underline{A}^T \cdot \Pi^{*T} \\ &= \underline{A} \cdot \underline{R}_{RP}^* \cdot \underline{A}^{*T}\end{aligned}$$

- averaged noise free correlation matrix:

$$\begin{aligned}\frac{\underline{R}_{ss} + \underline{R}_{ss,B}}{2} &= \underline{A} \cdot \frac{\underline{R}_{RP} + \underline{R}_{RP}^*}{2} \cdot \underline{A}^{*T} \\ &= \underline{A} \cdot \underbrace{\mathrm{Re}(\underline{R}_{RP})}_{\text{reduced correlations}} \cdot \underline{A}^{*T}\end{aligned}$$

- signal processing point of view:

$$\underline{R}_{ss,B} = \Pi \cdot E\{\underline{s}^* \cdot \underline{s}^T\} \cdot \Pi^{*T} = \Pi \cdot \underline{R}_{ss}^* \cdot \Pi^{*T}$$

$\Rightarrow$ compute an average (sample) correlation matrix from $\underline{R}_{ee}$ and

$$\underline{R}_{ee,B} = \Pi \cdot \underline{R}_{ee}^* \cdot \Pi^{*T}$$

```
function Ravg = forwardbackwardaveraging(Ree)
%FORWARDBACKWARDAVERAGING forward backward averaging
%   requires a centrosymmetric array
%   Ravg: averaged covariance matrix
%   Ree: covariance matrix

Ravg = (Ree+flipud(fliplr(conj(Ree))))/2;

end
```