# Conventional Beamformer

# Conventional Beamformer

- maximize the antenna gain $g(\varphi)$ for a certain direction of arrival $\varphi$

- Schwarz inequality:

$$g(\varphi) = |\underline{w}^{*T} \cdot \underline{a}(\varphi)|^2 \leq ||\underline{w}||^2 \; ||\underline{a}(\varphi)||^2$$

equality, i.e., maximum for

$$\underline{w} \sim \underline{a}(\varphi)$$

$$\Rightarrow \underline{w}^* = \frac{\underline{a}^*(\varphi)}{||\underline{a}(\varphi)||^2}$$

- power at the output of the beamforming network:

$$P_{conv}(\varphi) = \underline{w}^{*T} \cdot \underline{R}_{ee} \cdot \underline{w}$$

$$= \frac{\underline{a}^{*T}(\varphi) \cdot \underline{R}_{ee} \cdot \underline{a}(\varphi)}{\underline{a}^{*T}(\varphi) \cdot \underline{a}(\varphi)}$$

- corresponds to
  - (spatial) matched filter
  - correlator
  - maximal ratio combining

ULA

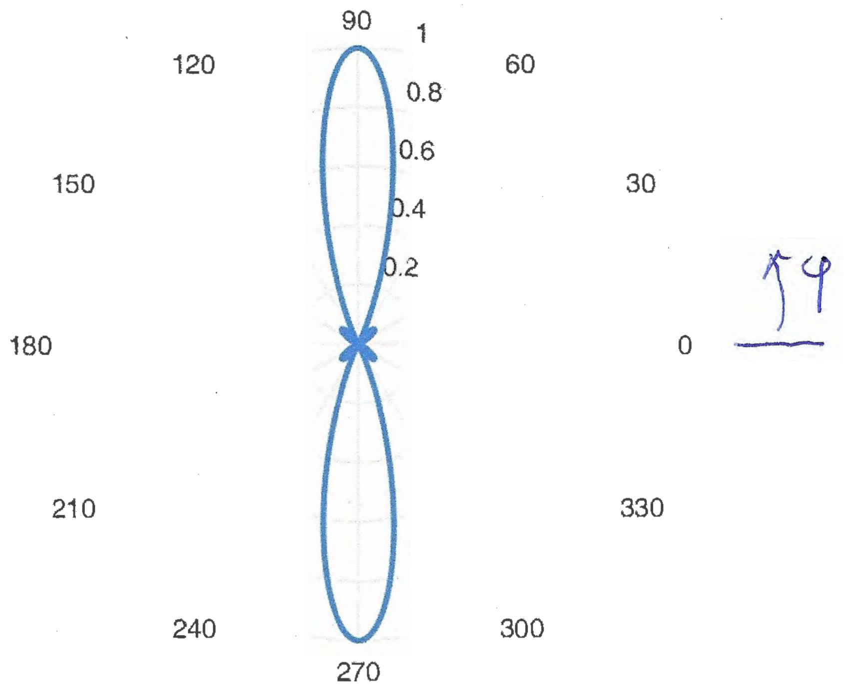
N = 4

d = 0,5 λ

φ = 90°

conventional beamformer

Antenna Pattern



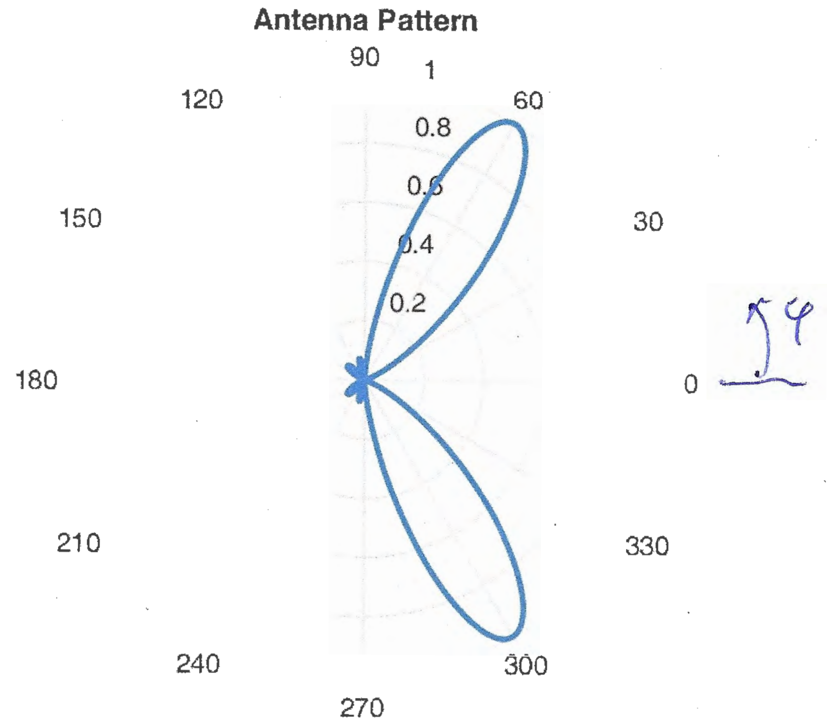
ULA

N=4

d=0,5λ

φ=60°

conventional beamformer



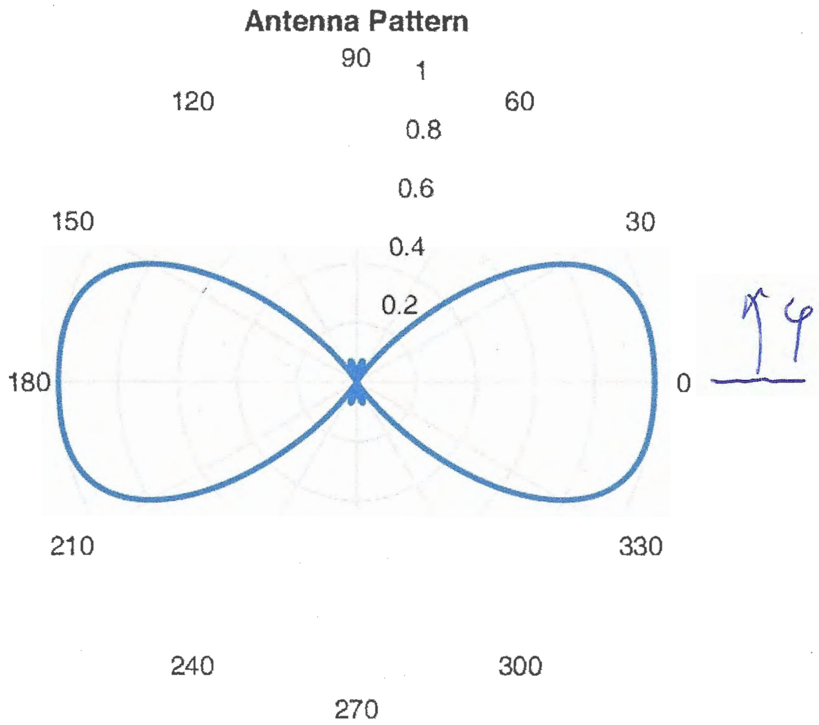
ULA

N=4

d=0,5λ

φ=0°

conventional beamformer



# Beamscan Algorithm

- idea:
  the power $P_{conv}(\varphi)$ at the output of the beamforming network should be large whenever $\varphi$ coincides with the direction of arrival $\varphi_k$ of a source k

  $\Rightarrow$ search for the (local) maxima of the spectrum

$$P_{conv}(\varphi) = \frac{\underline{a}^{*T}(\varphi) \cdot \underline{R}_{ee} \cdot \underline{a}(\varphi)}{\underline{a}^{*T}(\varphi) \cdot \underline{a}(\varphi)}$$

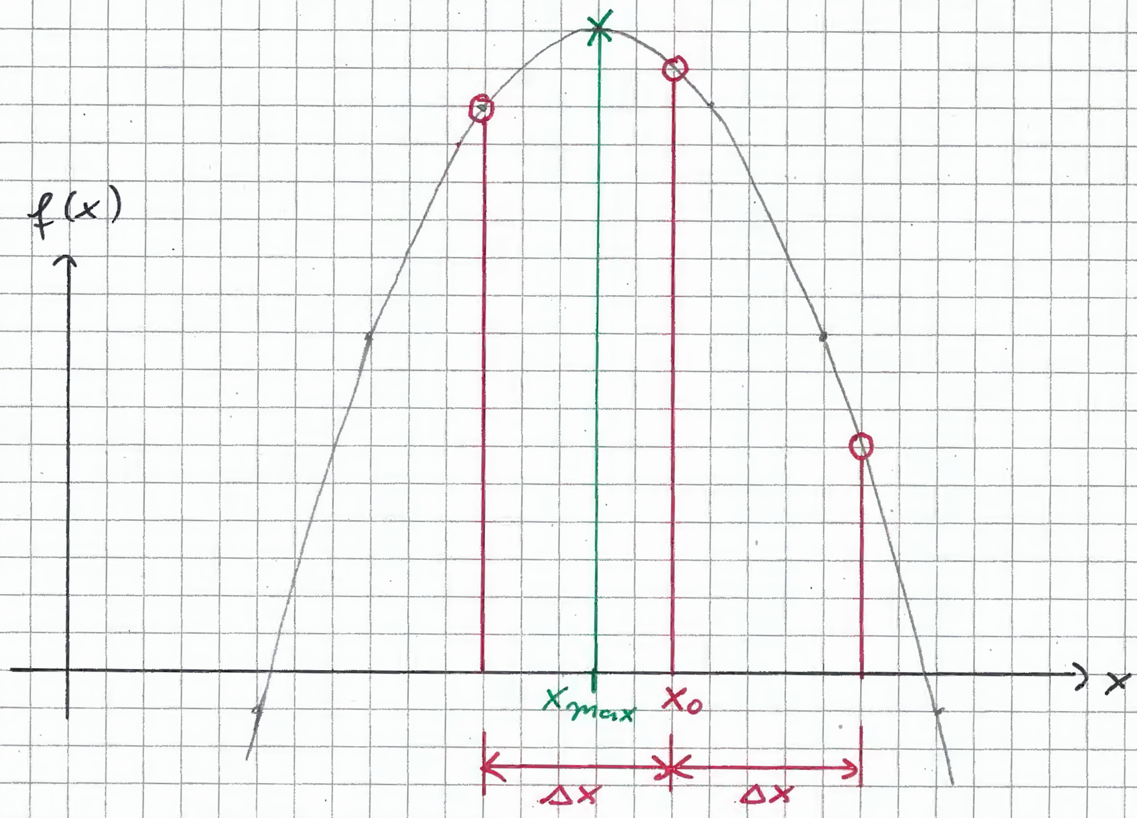
- is equivalent to deterministic maximum likelihood (DML) for the single source case

- challenges in the multiple source case:
  - beamwidth
    $\Rightarrow$ limited resolution
  - side lobes
    $\Rightarrow$ use window functions in the spatial domain

```
function phi = conventional(Ree, K, d, manifold, linear, varargin)
%CONVENTIONAL conventional beamformer
%   phi: directions of arrival
%   Ree: correlation matrix
%   K: number of sources
%   d: distance of antenna elements normalized to the wavelength
%   manifold: antenna array
%   linear: limit search range to pi for linear arrays

samples = 1001; % number of samples for plotting the spectrum

N = length(Ree); % number of antenna elements

% Using the array manifold vector the spectrum is computed as a'*Ree*a/(a'*a).
% Passing a vector of directions of arrival one obtains a
% vector of samples of the spectrum.
spectrum = @(phi) real(sum(conj(manifold(phi,N,d)).*(Ree*manifold(phi,N,d)),1)...
    ./sum(conj(manifold(phi,N,d)).*manifold(phi,N,d),1));

% determine all local maxima
phi = maxsearch(spectrum, linear);

% select the K largest local maxima
K = min([K, numel(phi)]);
phi = phi(1:K);

% plot the normalized spectrum
figure;
psi = linspace(0, 1, samples)*2*pi;
h = polar(psi, spectrum(psi)./max(spectrum(psi)), '-');
set(h, 'LineWidth', 2);
set(h, 'Color', 'b');
hold on;
for k = 1:K % plot the estimated directions of arrival
    h = polar([phi(k) phi(k)], [0 1], '-');
    set(h, 'LineWidth', 2);
    set(h, 'Color', 'r');
end
title('Spectrum');

end
```

# Parabolic Interpolation

parabola: $f(x) = a x^2 + b x + c$




The maximum $x_{max}$ can be computed from the maximum sample $x_0$ and its two neighbours.

① $f(x_0 - \Delta x) = a(x_0 - \Delta x)^2 + b(x_0 - \Delta x) + c$

② $f(x_0) = a x_0^2 + b x_0 + c$

③ $f(x_0 + \Delta x) = a(x_0 + \Delta x)^2 + b(x_0 + \Delta x) + c$

③ − 2 ② + ①

$$f(x+\Delta x) - 2f(x) + f(x-\Delta x) = 2a\,\Delta x^2$$

$$\Rightarrow \quad a = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{2\,\Delta x^2}$$

③ − ①

$$f(x+\Delta x) - f(x-\Delta x) = 4a\,x\,\Delta x + 2b\,\Delta x$$

$$\Rightarrow \quad b = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\,\Delta x} - 2ax$$

maximum:

$$x_{max} = -\frac{b}{2a}$$

$$= x - \frac{f(x+\Delta x) - f(x-\Delta x)}{4a\,\Delta x}$$

$$= x - \frac{f(x+\Delta x) - f(x-\Delta x)}{2f(x+\Delta x) - 4f(x) + 2f(x-\Delta x)}\,\Delta x$$

```
function phi = maxsearch(f, linear)
%MAXSEARCH determine all local maxima
%   phi: local maxima in nonincreasing order
%   f: function
%   linear: limit search range to pi for linear arrays

samples = 1001; % number of samples for the grid search

if linear
    limit = pi;
else
    limit = 2*pi;
end

stepsize = limit/(samples-1);

K = 0; % number of local maxima
for psi = linspace(0, 1, samples)*limit
    if (f(psi)>f(psi-stepsize) & f(psi)>f(psi+stepsize))...
           | (f(psi)==f(psi-stepsize)) % local maximum found
        K = K+1;
        phi(K) = psi;
    end
end

% parabolic interpolation
phi = phi(:)-(f(phi+stepsize)-f(phi-stepsize))...
    ./(2*f(phi+stepsize)-4*f(phi)+2*f(phi-stepsize))*stepsize;

% nonincreasing order
[value, index] = sort(f(phi), 'descend');
phi = phi(index); % resort

end
```

## Example: Uniform Linear Array

- direction of arrival characterised by $\beta_x$

- steering factor:

$$\underline{a}_n = e^{-j\frac{N+1}{2} d\beta_x} \; e^{jnd\beta_x}$$

$\Rightarrow$ weights

$$\underline{w}_n^* = e^{j\frac{N+1}{2} d\beta_x} \; e^{-jnd\beta_x}$$

- signal at the output of the beamforming network:

$$\underline{r} = e^{j\frac{N+1}{2} d\beta_x} \underbrace{\sum_n e^{-jnd\beta_x} \underline{e}_n}_{\text{corresponds to a Fourier transform}}$$

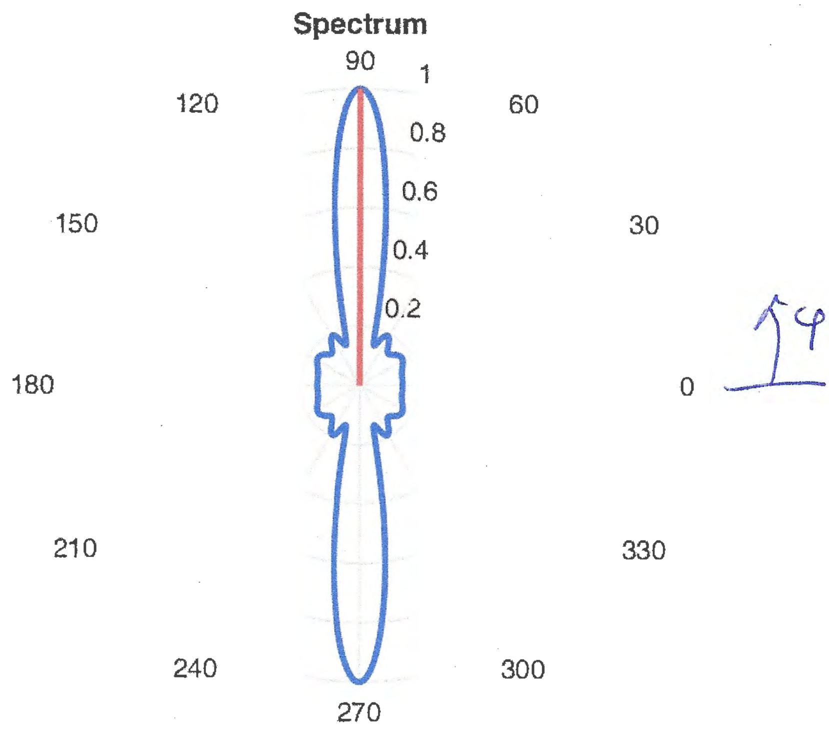
- the spectrum

$$P_{conv} = E\{|\underline{r}|^2\}$$

corresponds to the averaged periodograms

$\Rightarrow$ Bartlett's method

ULA
$N=6$
$d=0{,}5\lambda$
$\sigma^2=1$
$\varphi_1=90°$ power 1
Conventional




$\hat{\varphi}_1=90°$

ULA

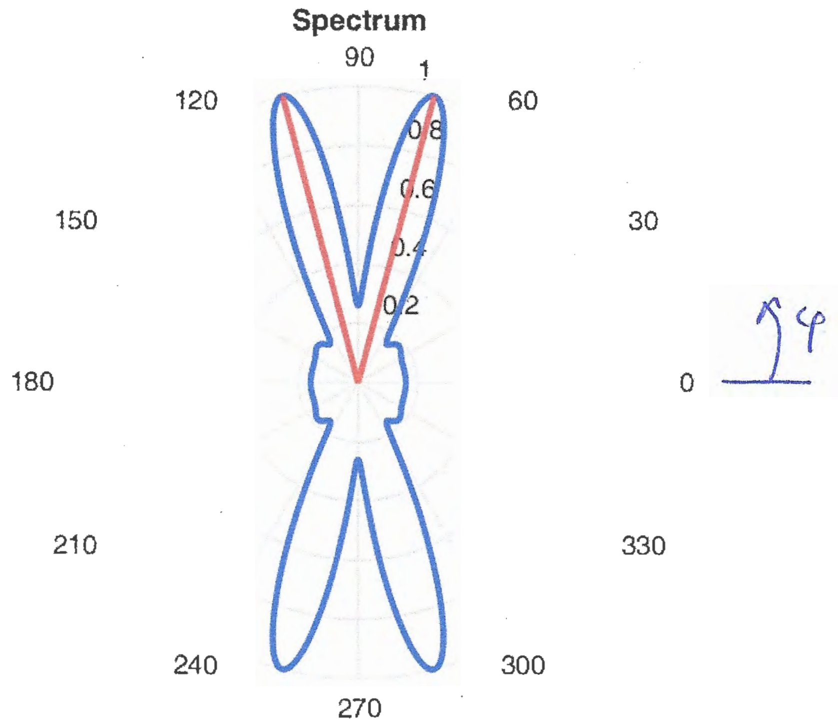
$N=6$

$d=0{,}5\lambda$

$\sigma^2=1$

$\left.\begin{array}{l}\varphi_1=75^\circ\\ \varphi_2=105^\circ\end{array}\right\}$ equal power 1

conventional




$\hat{\varphi}_1 = 104{,}77^\circ$

$\hat{\varphi}_2 = 75{,}29^\circ$

ULA

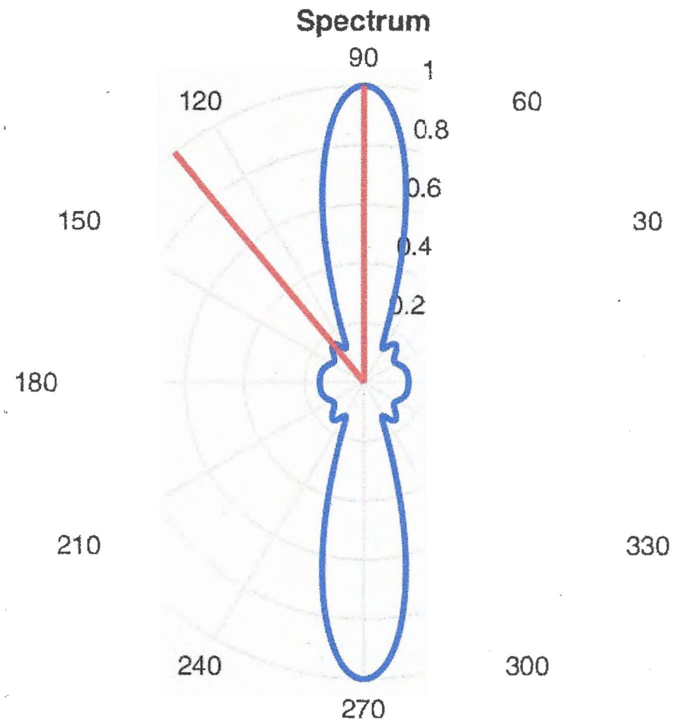
$N = 6$

$d = 0{,}5\lambda$

$\sigma^2 = 1$

$\left.\begin{array}{l} \varphi_1 = 83^\circ \\ \varphi_2 = 97^\circ \end{array}\right\}$ equal power 1

conventional





$\hat{\varphi}_1 = 90^\circ$

$(\hat{\varphi}_2 = 129{,}39^\circ)$

the two sources can not be resolved