Beamforming Network

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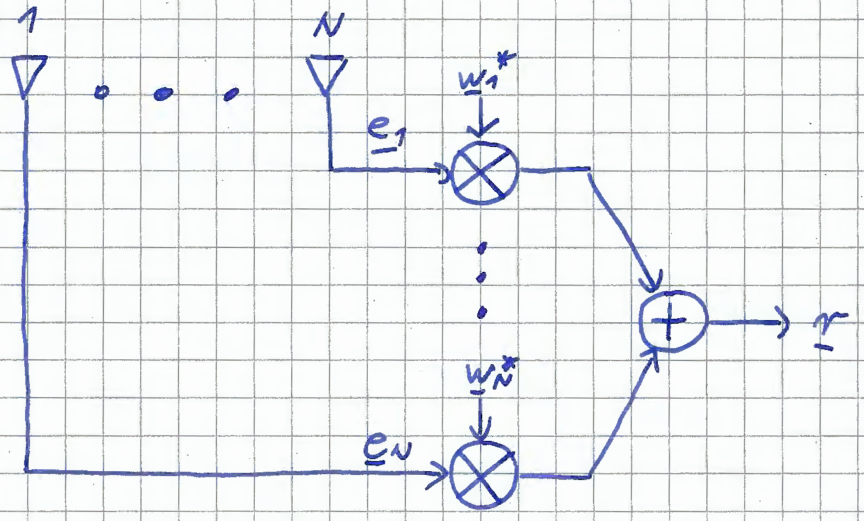


- weigths : $\underline{w}_n^*$
- weight vector:

$$\underline{w}^* = \begin{pmatrix} \underline{w}_1^* \\ \vdots \\ \underline{w}_N^* \end{pmatrix}$$

- idea:
  adjust the weights such that the phase shifts (steering factors) for a certain direction of arrival are compensated
  $\Rightarrow$ signals from this direction of arrival will superimpose constructively
- antenna array together with the beamforming network acts like a directional antenna

Received Signal

- received vector:

$$\underline{e} = \begin{pmatrix} \underline{e}_1 \\ \vdots \\ \underline{e}_N \end{pmatrix}$$

- signal at the output of the beamforming network:

$$\underline{r} = \sum_n \underline{w}_n^* \, \underline{e}_n = \underline{w}^{*T} \cdot \underline{e}$$

- (instantaneous) power at the output of the beamforming network:

$$P_r = |\underline{r}|^2 = |\underline{w}^{*T} \cdot \underline{e}|^2 \underset{\uparrow}{\leq} \|\underline{w}\|^2 \, \|\underline{e}\|^2$$

Schwarz inequality

- passive beamforming network:

$$P_r \leq \|\underline{w}\|^2 \, \|\underline{e}\|^2 \leq \|\underline{e}\|^2 = P$$

$$\Rightarrow \|\underline{w}\|^2 \leq 1$$

- in the following:

$$\|\underline{w}\|^2 = 1 \qquad \text{"lossless"}$$

Antenna Gain

- consider a single source:
  - single direction of arrival φ
  - single array manifold vector $\underline{a}(\varphi)$
  - single source signal s_{RP}
- received vector:

$$\underline{s} = \underline{a}(\varphi) \cdot s_{RP}$$

$$\Rightarrow r = \underbrace{\underline{w}^{*T} \cdot \underline{a}(\varphi)}_{\substack{\underline{F}(\varphi) \\ \text{array factor}}} \cdot s_{RP}$$

- antenna gain:

$$g(\varphi) = |\underline{F}(\varphi)|^2$$

$$= |\underline{w}^{*T} \cdot \underline{a}(\varphi)|^2$$

$$= \underline{w}^{*T} \cdot \underbrace{\underline{a}(\varphi) \cdot \underline{a}^{*T}(\varphi)}_{\text{positive semidefinite matrix}} \cdot \underline{w} \geq 0$$

- antenna pattern:

antenna gain normalized to its maximum value

$$C(\varphi) = \frac{g(\varphi)}{g(\varphi_{max})} = \frac{\underline{w}^{*T} \cdot \underline{a}(\varphi) \cdot \underline{a}^{*T}(\varphi) \cdot \underline{w}}{\underline{w}^{*T} \cdot \underline{a}(\varphi_{max}) \cdot \underline{a}^{*T}(\varphi_{max}) \cdot \underline{w}}$$

plotted as a function of the direction of arrival φ

Example: Uniform Linear Array

- array factor:

$$\underline{F} = \underline{w}^{*T} \cdot \underline{a} = \sum_n \underline{w}_n^* \, \underline{a}_n$$

- Using

$$\underline{a}_n = \underline{\varepsilon}^{-\frac{N-1}{2}} \, \underline{\varepsilon}^{n}$$

one obtains:

$$\underline{F}(\underline{\varepsilon}) = \underline{\varepsilon}^{-\frac{N-1}{2}} \underbrace{\sum_n \underline{w}_n^* \, \underline{\varepsilon}^{n}}_{\text{corresponds to a z-transform}}$$

$\Rightarrow$ Design of beamforming networks is a filter design problem (spatial filter).

Aliasing

- $$\underline{\varepsilon} = e^{j d \beta_x}$$

and consequently also
- the array factor $\overline{\underline{I}}$,
- the antenna gain g and
- the antenna pattern G'

are periodic functions of β_x.

- Periodic repetitions of the main lobe of the antenna pattern G' are called grating lobes.

- period:

$$-\pi \leq d\,\beta_x \leq +\pi$$

- visible region:

$$-\frac{2\pi}{\lambda} \leq \beta_x \leq +\frac{2\pi}{\lambda}$$

$\Rightarrow$ no visible periodicity if

$$d\,\frac{2\pi}{\lambda} \leq +\pi$$

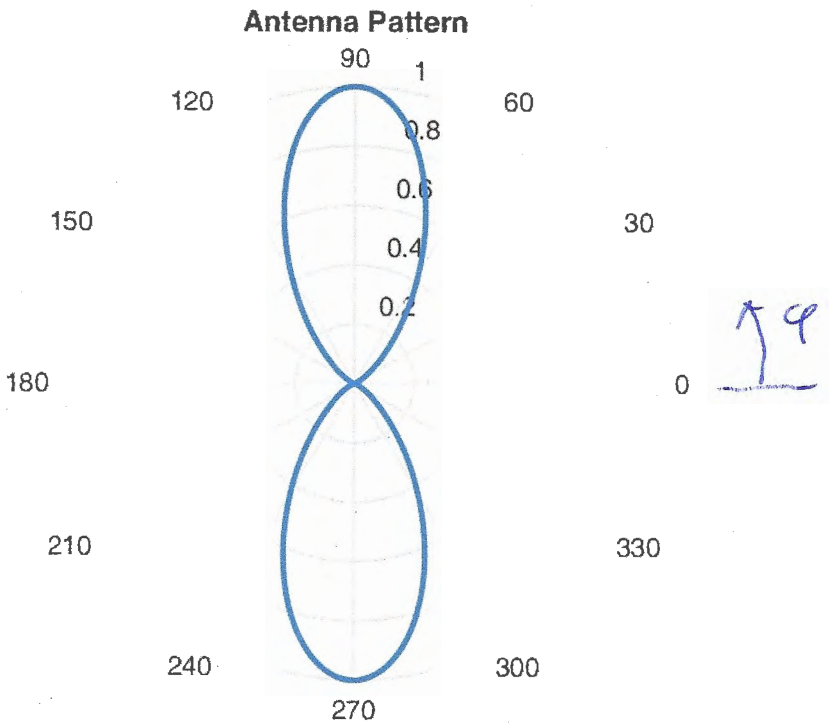
$$d \leq \frac{\lambda}{2}$$

ULA

$N = 4$

$d = 0{,}25\lambda$

$\underline{w}^* = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$

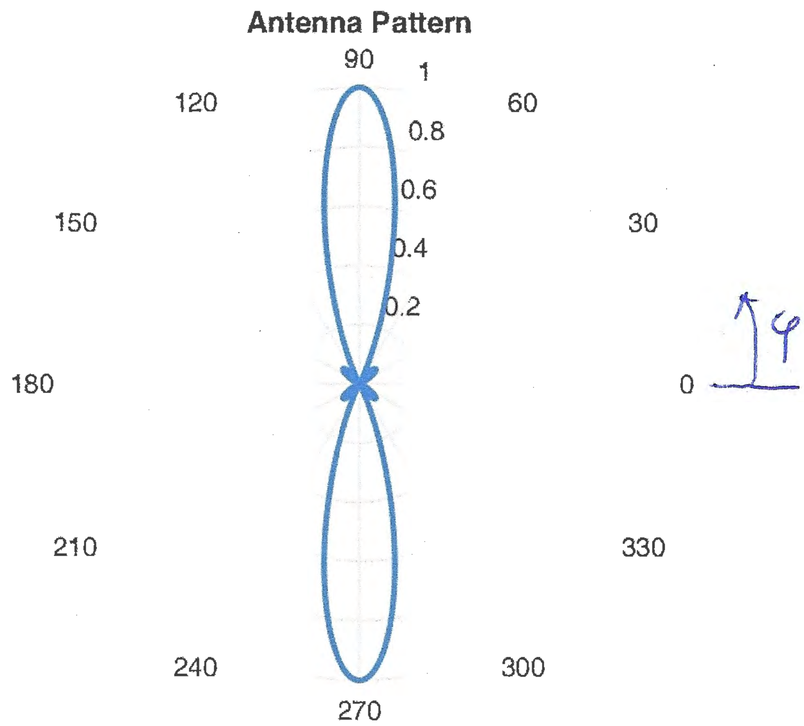


ULA

$N = 4$

$d = 0{,}5\lambda$

$\underline{w}^* = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$



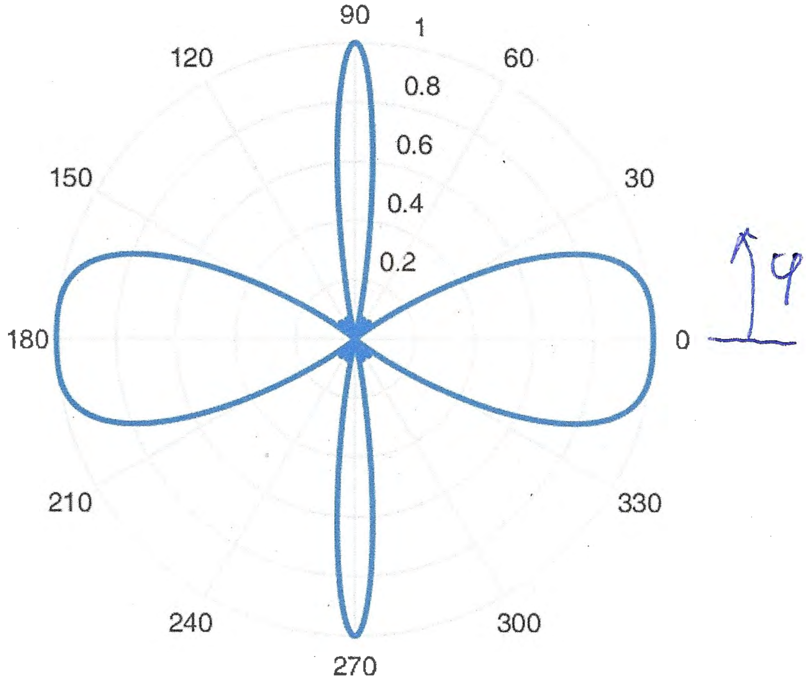
ULA

$N=4$

$d = 1\lambda$

$\underline{w}^* = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

Antenna Pattern



Power at the Output of the Beamforming Network

several sources and noise

$$\Rightarrow \quad P_r = E\{|\underline{r}|^2\}$$

$$= E\{\underline{w}^{*T} \cdot \underline{e} \cdot \underline{e}^{*T} \cdot \underline{w}\}$$

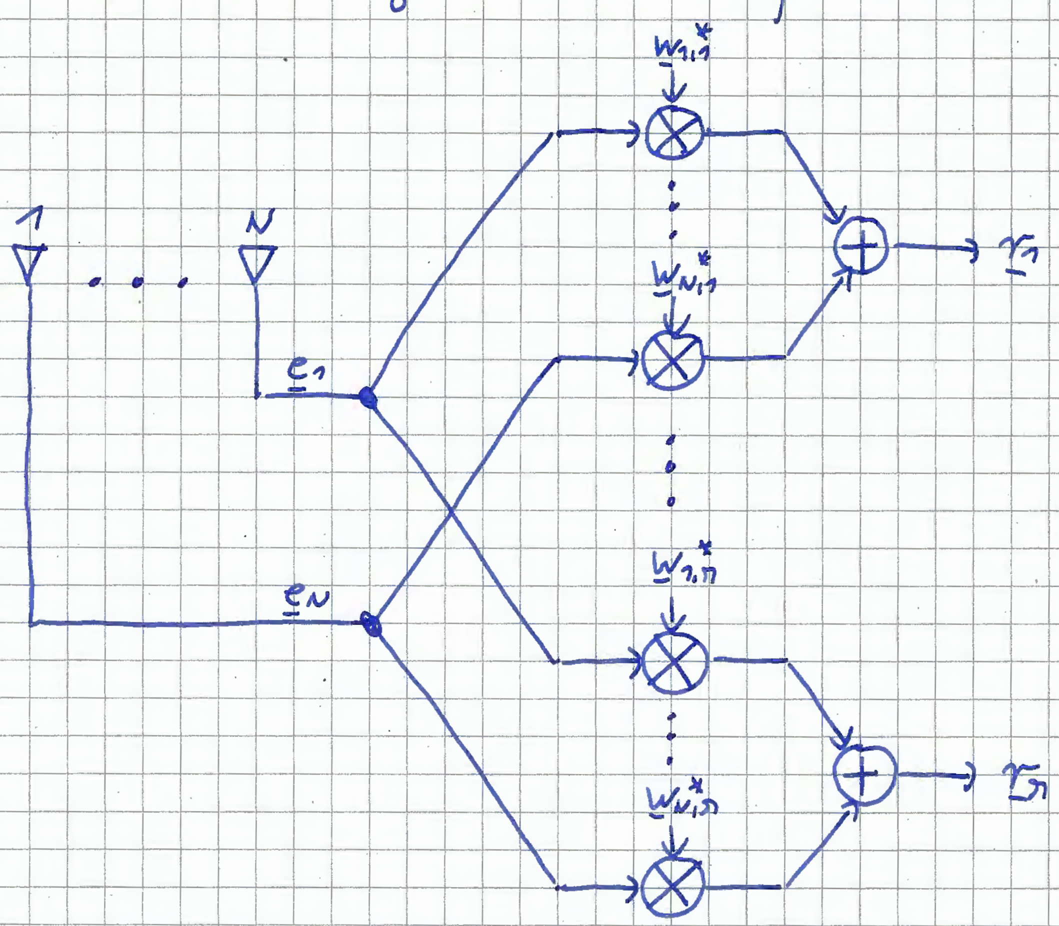
$$= \underline{w}^{*T} \cdot E\{\underline{e} \cdot \underline{e}^{*T}\} \cdot \underline{w}$$

$$= \underline{w}^{*T} \cdot \underline{R}_{ee} \cdot \underline{w}$$

All the relevant information about the received signals is contained in the correlation matrix $\underline{R}_{ee}$.

Beamspace

- use M beamforming networks in parallel:



- received vector in beamspace:

$$\underbrace{\begin{pmatrix} \underline{r}_1 \\ \vdots \\ \underline{r}_M \end{pmatrix}}_{\underline{r}} = \underbrace{\begin{pmatrix} \underline{w}_{1,1}^* & \cdots & \underline{w}_{N,1}^* \\ \vdots & & \vdots \\ \underline{w}_{1,M}^* & \cdots & \underline{w}_{N,M}^* \end{pmatrix}}_{\underline{W}^{*T}} \cdot \underbrace{\begin{pmatrix} \underline{e}_1 \\ \vdots \\ \underline{e}_N \end{pmatrix}}_{\underline{e}}$$

weight matrix

- invertible weight matrix:
  $\Rightarrow$ one-to-one mapping, no loss of information

- Fourier matrix as a weight matrix
  $\Rightarrow$ Butler matrix, low cost implementation similar to fast Fourier transform