Multiple Signal Classification (MUSIC)

Subspace Methods

- Exploit the structure of
  - the noise subspace $\underline{U}_n$ (MUSIC) or
  - the signal subspace $\underline{U}_s$ (ESPRIT).

- Both the noise subspace $\underline{U}_n$ and the signal subspace $\underline{U}_s$ must exist.
  $\Rightarrow$ Number K of sources limited by the number N of antenna elements:

$$0 < K < N .$$

- Performance limited by the ability to accurately determine the noise subspace $\underline{U}_n$ and the signal subspace $\underline{U}_s$, i.e., by the averaging required to estimate the correlation matrix $\underline{R}_{ee}$.

MUSIC

- idea:

  The array manifold vectors $\underline{a}_k$ of the sources are orthogonal to the noise subspace $\underline{U}_n$.

  $\Rightarrow$ Search for array manifold vectors $\underline{a}(\varphi)$ being orthogonal to the noise subspace $\underline{U}_n$:

$$\underline{a}^{*T}(\varphi) \cdot \underline{U}_n = 0 .$$

- in reality not perfect orthogonality

  $\Rightarrow$ Search for the (local) minima of the null spectrum

$$Q_{MUSIC}(\varphi) = \| \underline{a}^{*T}(\varphi) \cdot \underline{U}_n \|^2$$

$$= \underline{a}^{*T}(\varphi) \cdot \underline{U}_n \cdot \underline{U}_n^{*T} \cdot \underline{a}(\varphi) .$$

alternatively (no difference for omnidirectional antenna elements):

$$Q_{MUSIC}(\varphi) = \frac{\underline{a}^{*T}(\varphi) \cdot \underline{U}_n \cdot \underline{U}_n^{*T} \cdot \underline{a}(\varphi)}{\underline{a}^{*T}(\varphi) \cdot \underline{a}(\varphi)}$$

```
function phi = music(Ree, K, d, manifold, linear, varargin)
%MUSIC multiple signal classification
%   phi: directions of arrival
%   Ree: correlation matrix
%   K: number of sources
%   d: distance of antenna elements normalized to the wavelength
%   manifold: antenna array
%   linear: limit search range to pi for linear arrays

samples = 1001; % number of samples for plotting the null spectrum

N = length(Ree); % number of antenna elements

% eigendecomposition
[U, L] = eig((Ree+Ree')/2, 'vector'); % make sure that the matrix is Hermitian
[L, index] = sort(L, 'descend');
U = U(:,index);
Un = U(:,K+1:N); % noise subspace

% Using the array manifold vector the null spectrum is computed as (a'*Un*Un'*a)/
(a'*a)
% Passing a vector of directions of arrival one obtains a
% vector of samples of the null spectrum.
nullspectrum = @(phi) real(sum(conj(manifold(phi,N,d)).*(Un*Un'*manifold(phi,N,d)),
1)...
    ./sum(conj(manifold(phi,N,d)).*manifold(phi,N,d),1));

% determine all local minima
phi = minsearch(nullspectrum, linear);

% select the K smallest local minima
K = min([K, numel(phi)]);
phi = phi(1:K);

% plot the normalized null spectrum
figure;
psi = linspace(0, 1, samples)*2*pi;
h = polar(psi, nullspectrum(psi)./max(nullspectrum(psi)), '-');
set(h, 'LineWidth', 2);
set(h, 'Color', 'b');
hold on;
for k = 1:K % plot the estimated directions of arrival
    h = polar([phi(k) phi(k)], [0 1], '-');
    set(h, 'LineWidth', 2);
    set(h, 'Color', 'r');
end
title('Null Spectrum');

end
```

```
function phi = minsearch(f, linear)
%MINSEARCH determine all local minima
%   phi: local minima in nondecreasing order
%   f: function
%   linear: limit search range to pi for linear arrays

samples = 1001; % number of samples for the grid search

if linear
    limit = pi;
else
    limit = 2*pi;
end

stepsize = limit/(samples-1);

K = 0; % number of local minima
for psi = linspace(0, 1, samples)*limit
    if (f(psi)<f(psi-stepsize) & f(psi)<f(psi+stepsize))...
            | (f(psi)==f(psi-stepsize)) % local minimum found
        K = K+1;
        phi(K) = psi;
    end
end

% parabolic interpolation
phi = phi(:)-(f(phi+stepsize)-f(phi-stepsize))...
    ./(2*f(phi+stepsize)-4*f(phi)+2*f(phi-stepsize))*stepsize;

% nondecreasing order
[value, index] = sort(f(phi), 'ascend');
phi = phi(index); % resort

end
```

ULA

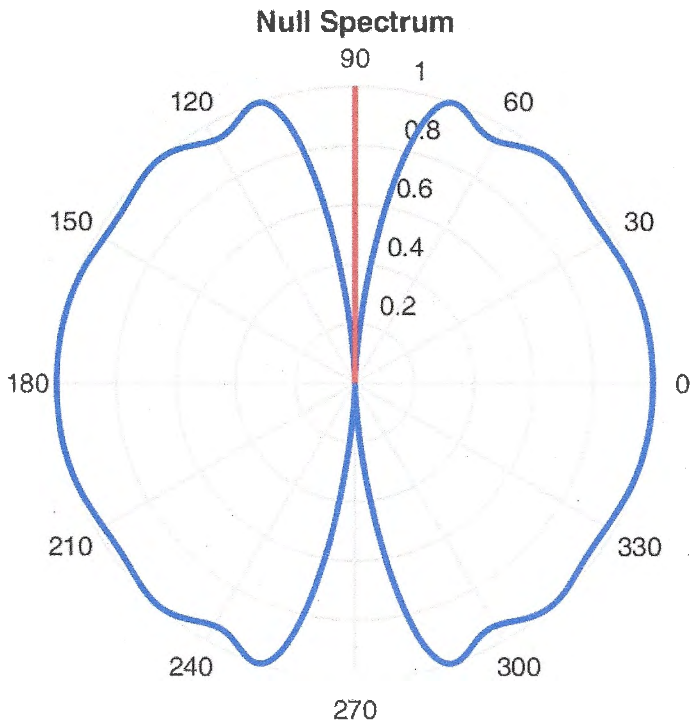
$N=6$

$d=0{,}5\lambda$

$\sigma^2=1$

$\varphi_1=90^\circ$ power 1

music



φ

$\tilde{\varphi}_1=90^\circ$

ULA

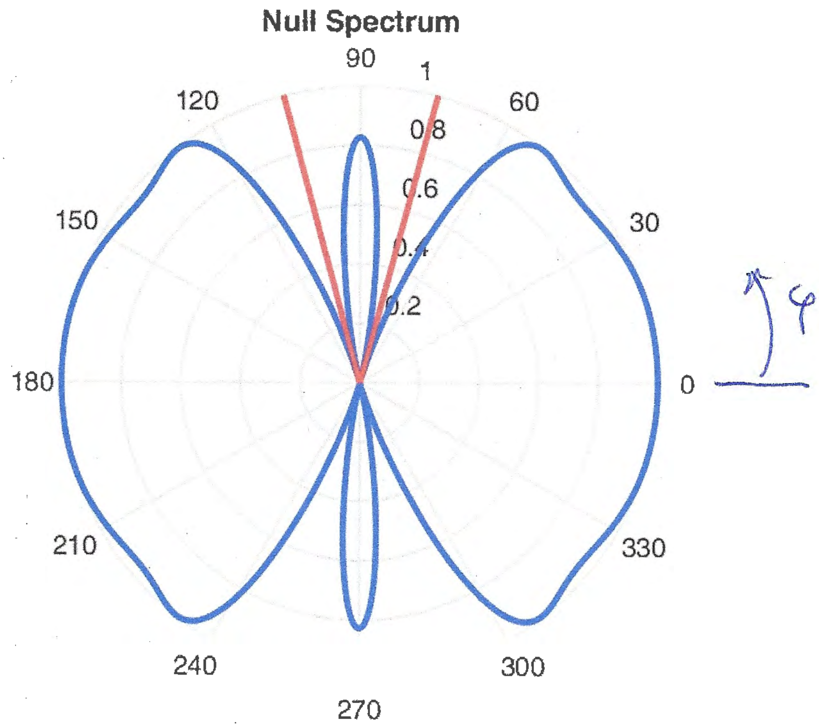
$N = 6$

$d = 0{,}5\lambda$

$\sigma^2 = 1$

$$\left.\begin{array}{l} \varphi_1 = 75^\circ \\ \varphi_2 = 105^\circ \end{array}\right\} \text{equal power } 1$$

music



$\hat{\varphi}_1 = 105{,}000^\circ$

$\hat{\varphi}_2 = 75{,}000^\circ$

ULA

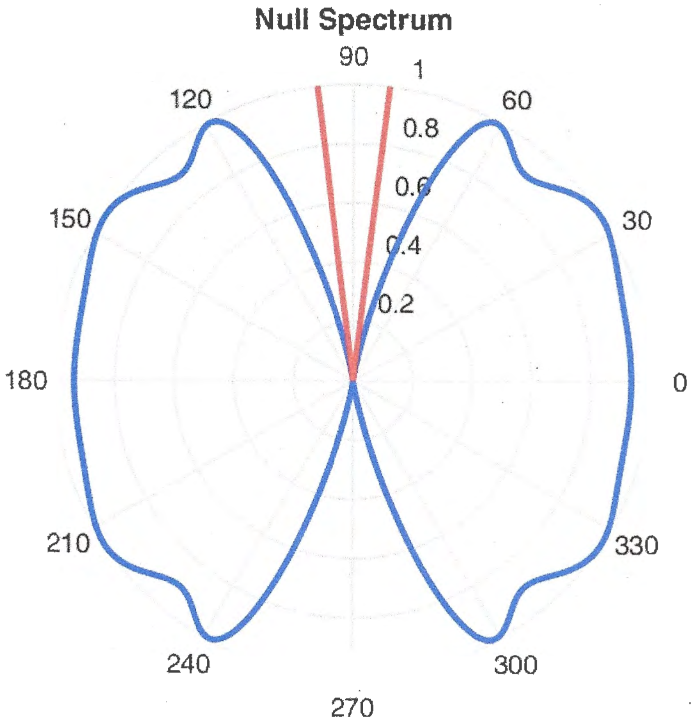
$N = 6$

$d = 0{,}5\lambda$

$\sigma^2 = 1$

$$\left.\begin{aligned} \varphi_1 &= 83^\circ \\ \varphi_2 &= 97^\circ \end{aligned}\right\} \text{ equal power } 1$$

music



φ

$\hat{\varphi}_1 = 83{,}001^\circ$

$\hat{\varphi}_2 = 96{,}998^\circ$