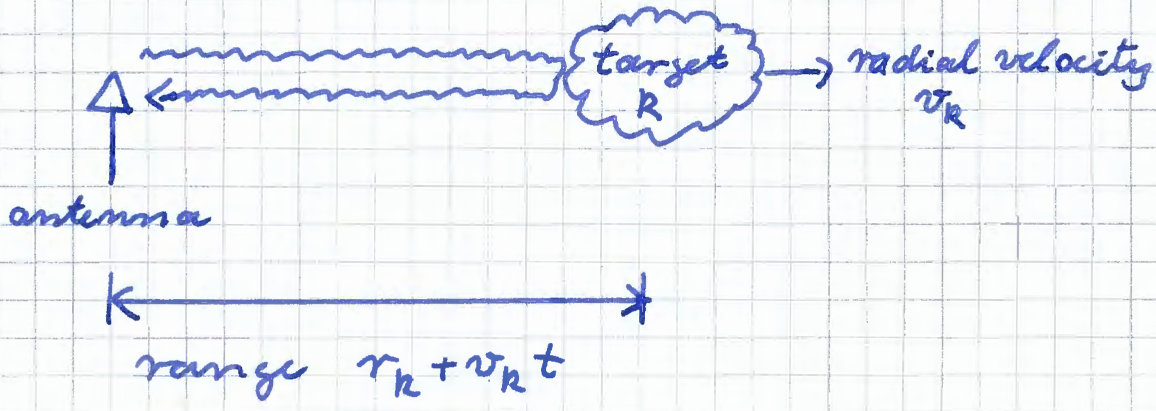
Introduction: FMCW Radar

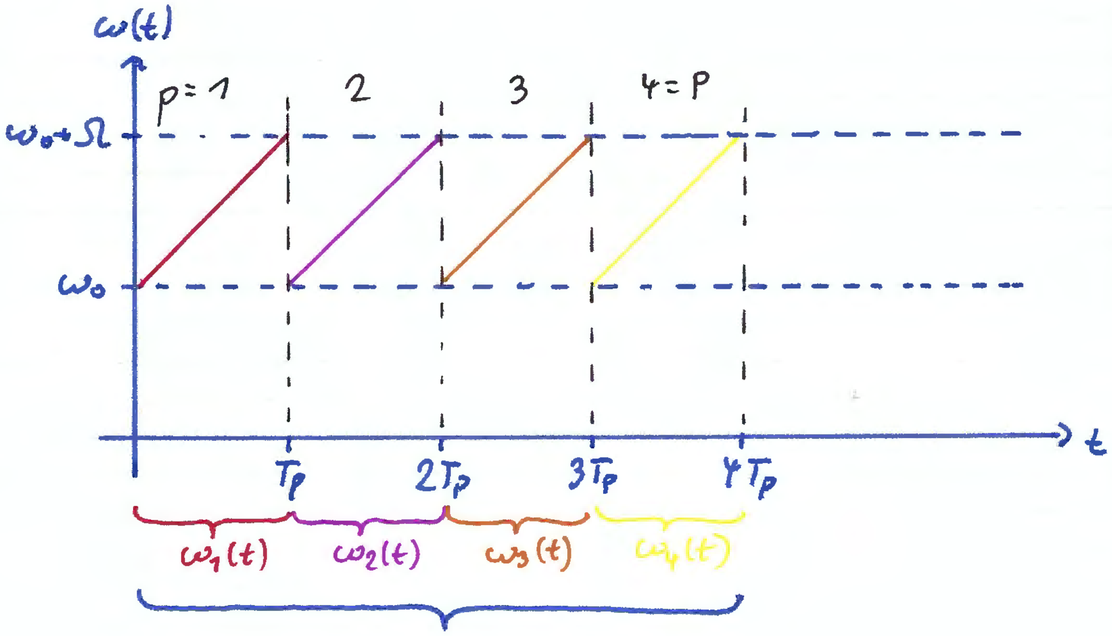
SISO Radar



delay:

$$t_k = \frac{2}{c_0}\left(r_k + v_k t\right)$$

Frequency Modulated Continuous Wave (FMCW) Radar



frame with $P=4$ pulses

Transmit linear frequency modulated pulses of duration T_p.

- instantaneous frequency in pulse p:

$$\omega_p(t) = \omega_0 + \frac{\Omega}{T_p}(t-pT_p)$$

- phase in pulse p:

$$\varphi_p(t) = \int \omega_p(t)\,dt = \omega_0(t-pT_p) + \frac{\Omega}{2T_p}(t-pT_p)^2$$

Noise Free Received Signal

noise free received signal in pulse p, several targets k:

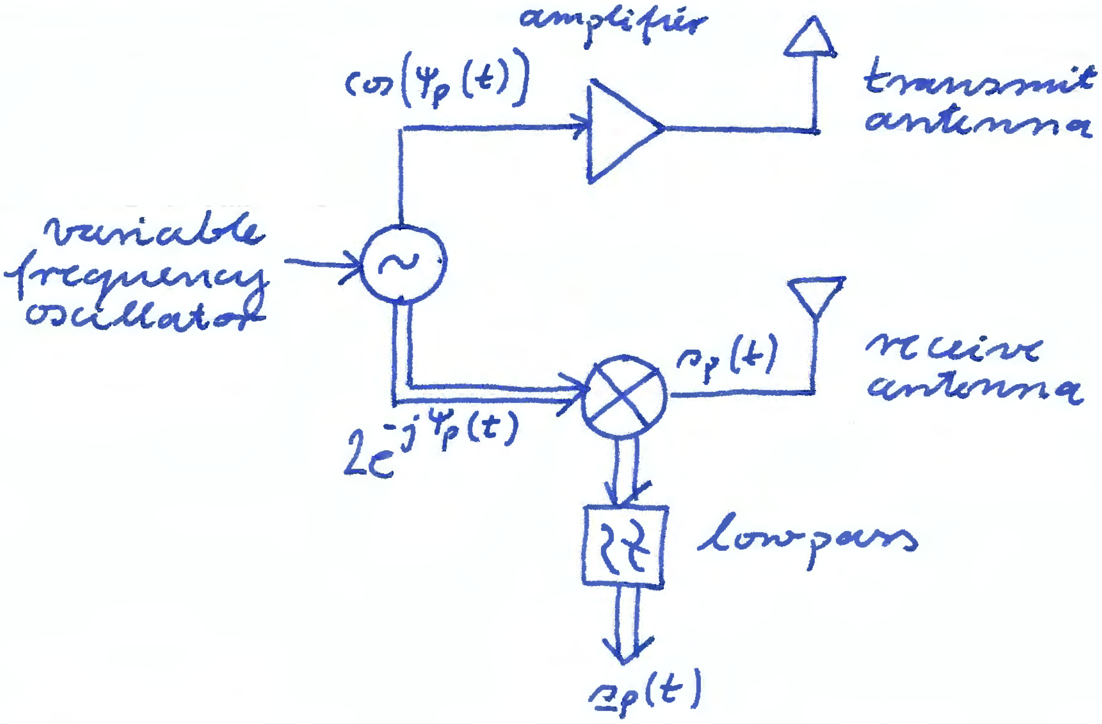
$$s_p(t) = \sum_k \underbrace{A_k}_{\text{amplitude of the target}} \cos\left(\underbrace{\psi_p(t-t_k)}_{\text{delay of the target}} + \underbrace{\alpha_k}_{\text{phase of the target}} \right)$$

$$= \sum_k A_k \cos\underbrace{\left(\omega_0 (t-t_k-pT_p) + \frac{\Omega}{2T_p}(t-t_k-pT_p)^2 + \alpha_k\right)}_{\psi_{p,k}(t)}$$

$$= \sum_k \frac{A_k}{2}\left(e^{j\psi_{p,k}(t)} + e^{-j\psi_{p,k}(t)}\right)$$

Stretch Processing

- low cost hardware architecture:



- mixing:

$$r_p(t)\cdot 2e^{-j\psi_p(t)} = \sum_k \frac{A_k}{2}\left(e^{j\psi_{p,k}(t)} + e^{-j\psi_{p,k}(t)}\right)\cdot 2e^{-j\psi_p(t)}$$

$$= \sum_k A_k\Big(e^{j(\psi_{p,k}(t)-\psi_p(t))} + \underbrace{e^{j(-\psi_{p,k}(t)-\psi_p(t))}}_{\text{high frequency}}\Big)$$

- lowpass filtering:

$$z_p(t) = \sum_k A_k\, e^{j(\psi_{p,k}(t)-\psi_p(t))}$$

Phase

- phase of target k in pulse p:

$$\varphi_{p,k}(t) - \varphi_p(t) = \omega_0 (t - t_k - pT_p) + \frac{\Omega}{2T_p}(t - t_k - pT_p)^2 + \alpha_k$$

$$- \omega_0 (t - pT_p) - \frac{\Omega}{2T_p}(t - pT_p)^2$$

$$= -\omega_0 t_k - \frac{\Omega}{T_p}(t - pT_p) t_k + \underbrace{\frac{\Omega}{2T_p} t_k^2}_{\text{small}} + \alpha_k$$

$$\approx -\omega_0 t_k - \frac{\Omega}{T_p}(t - pT_p) t_k + \alpha_k$$

$$\Big| \; t_k = \frac{2}{c_0}(r_k + v_k t)$$

$$= -\omega_0 \frac{2}{c_0} r_k - \omega_0 \frac{2}{c_0} v_k t$$

$$- \frac{\Omega}{T_p} \frac{2}{c_0} r_k (t - pT_p) - \underbrace{\frac{\Omega}{T_p} \frac{2}{c_0} v_k \underset{\substack{\approx \\ pT_p}}{t} (t - pT_p)}_{\substack{\text{small} \\ \text{(range-Doppler coupling)}}} + \alpha_k$$

$$\approx \underbrace{- \left(\omega_0 \frac{2}{c_0} v_k + \frac{\Omega}{T_p} \frac{2}{c_0} r_k \right)}_{\omega_k} (t - pT_p)$$

$$\underbrace{- \omega_0 \frac{2}{c_0} v_k}_{\omega_{D,k}} \; pT_p$$

$$\underbrace{- \omega_0 \frac{2}{c_0} r_k + \alpha_k}_{\text{constant}}$$

Range-Doppler Coupling

two views of the range-Doppler coupling $-\frac{\Omega}{T_p}\frac{2}{c_0}v_k\, p T_p\,(t-pT_p)$:

- range changes over slow time (range migration):

$$\Delta r_k = v_k\, p T_p$$

$\Rightarrow$ frequency shift ω_k changes

$$\Delta\omega_k = -\frac{\Omega}{T_p}\frac{2}{c_0}\Delta r_k = -\frac{\Omega}{T_p}\frac{2}{c_0}v_k\, p T_p$$

- instantaneous frequency of the transmitted signal changes over fast time (Doppler migration):

$$\Delta\omega_p = \frac{\Omega}{T_p}(t-pT_p)$$

$\Rightarrow$ Doppler frequency $\omega_{D,k}$ changes

$$\Delta\omega_{D,k} = -\Delta\omega_p\frac{2}{c_0}v_k = -\frac{\Omega}{T_p}(t-pT_p)\frac{2}{c_0}v_k$$

- range-Doppler coupling will be neglected in the following

- Doppler frequency:

$$\omega_{D,k} = -\omega_0 \frac{2}{c_0} v_k$$

$\Rightarrow$ compute the velocity from the phase shift inbetween the pulses:

$$v_k = -\frac{c_0}{2} \frac{\omega_{D,k}}{\omega_0}$$

- frequency shift:

$$\omega_k = \omega_{D,k} - \frac{\Omega}{T_p} \frac{2}{c_0} r_k$$

$$\approx -\frac{\Omega}{T_p} \frac{2}{c_0} r_k$$

$\Rightarrow$ compute the range from the frequency shift of the pulses:

$$r_k = -\frac{c_0}{2} \frac{T_p}{\Omega} (\omega_k - \omega_{D,k}) \approx -\frac{c_0}{2} \frac{T_p}{\Omega} \omega_k$$

- Velocity and range estimation are harmonic retrieval problems.

$\Rightarrow$ Same estimation techniques as for direction of arrival estimation can be applied.

- challenge: single sample / measurement

Sampling

- sampling in
  - fast time domain (lT) and
  - slow time domain (pT_p)

$$\underline{s}_p(lT + pT_p) = \sum_k \underline{A}_k \, e^{j\omega_{D,k} pT_p} \, e^{j\omega_k lT}$$

using $\underline{A}_k = A_k \, e^{j\left(-\omega_0 \frac{2}{c_0} r_k + \alpha_k\right)}$

- unambiguous Doppler frequency, velocity:

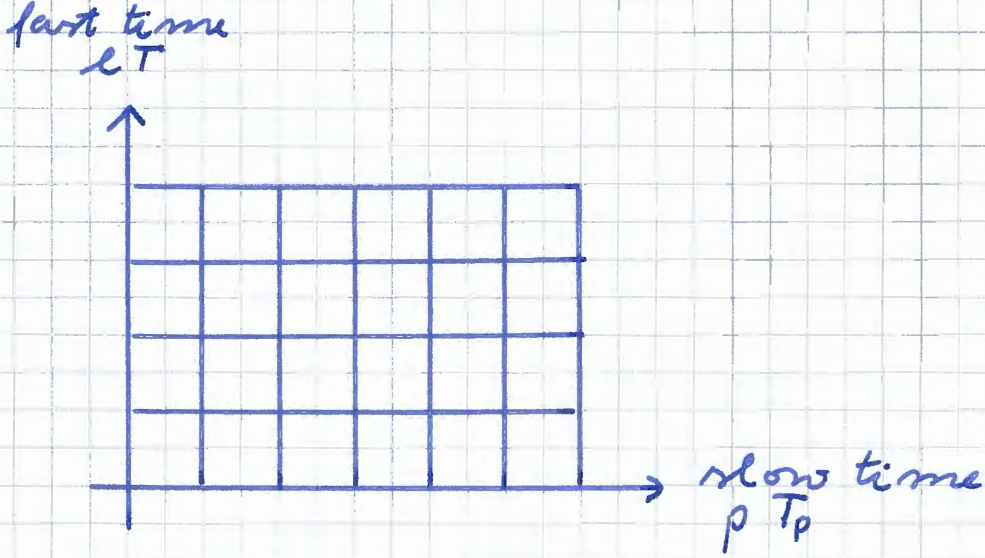
$$T_p < \frac{\pi}{\omega_{D\,max}} = \frac{\pi c_0}{2\omega_0 v_{max}}$$

- unambiguous frequency shift, range

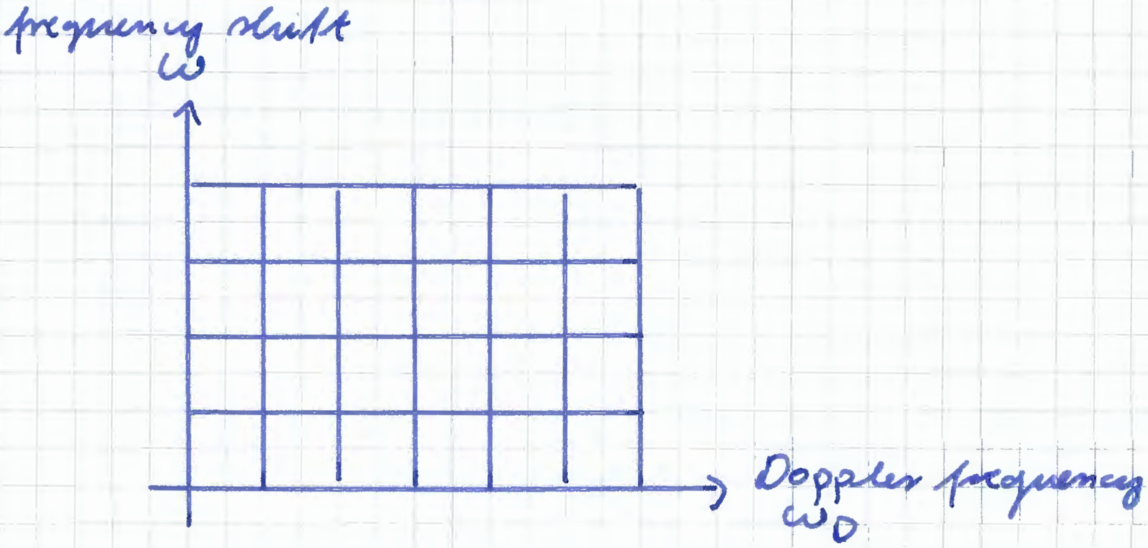
$$T < \frac{2\pi}{\omega_{max}} = \frac{\pi T_p c_0}{\Omega \, r_{max}}$$

Data Matrix

one column per pulse

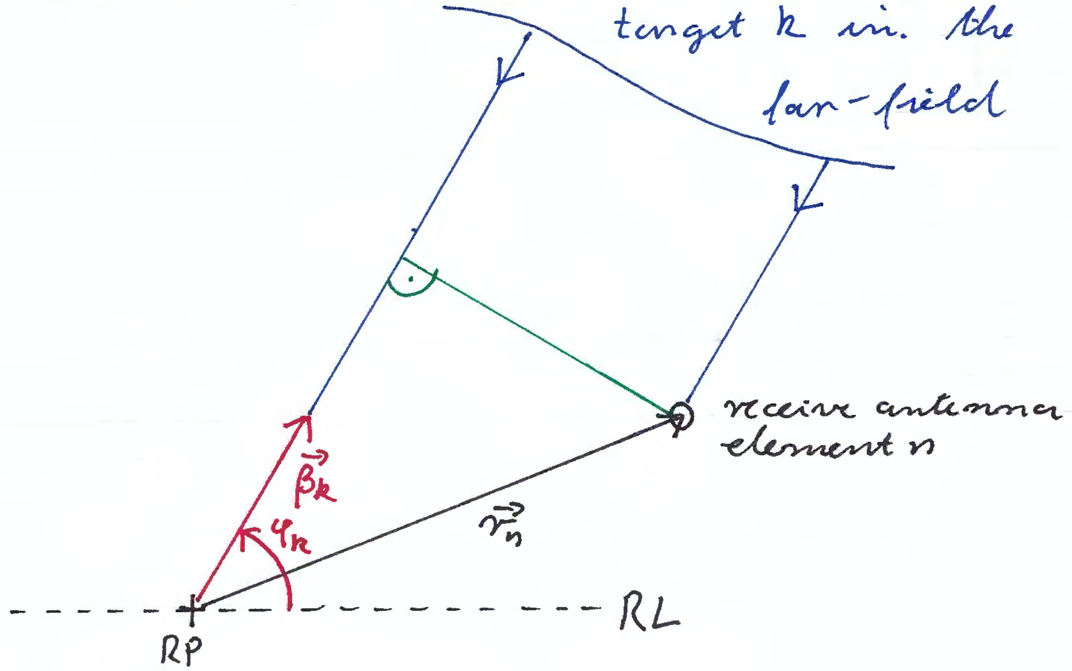


2D Fourier transform



SIMO Radar

- use N receive antenna elements for direction of arrival φ_k estimation

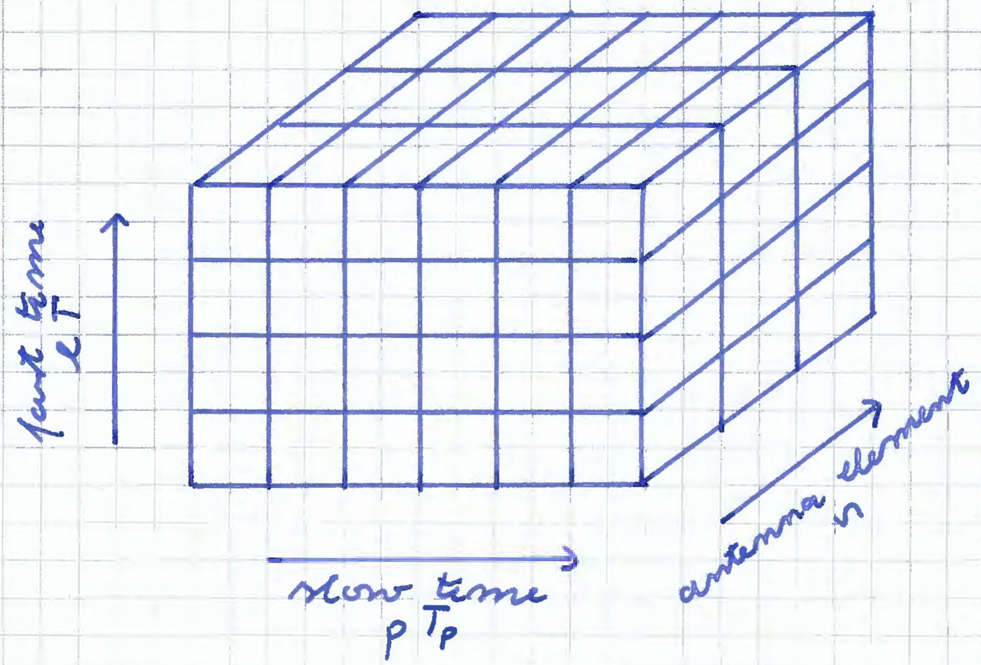


- phase of target k at antenna element n in pulse p:

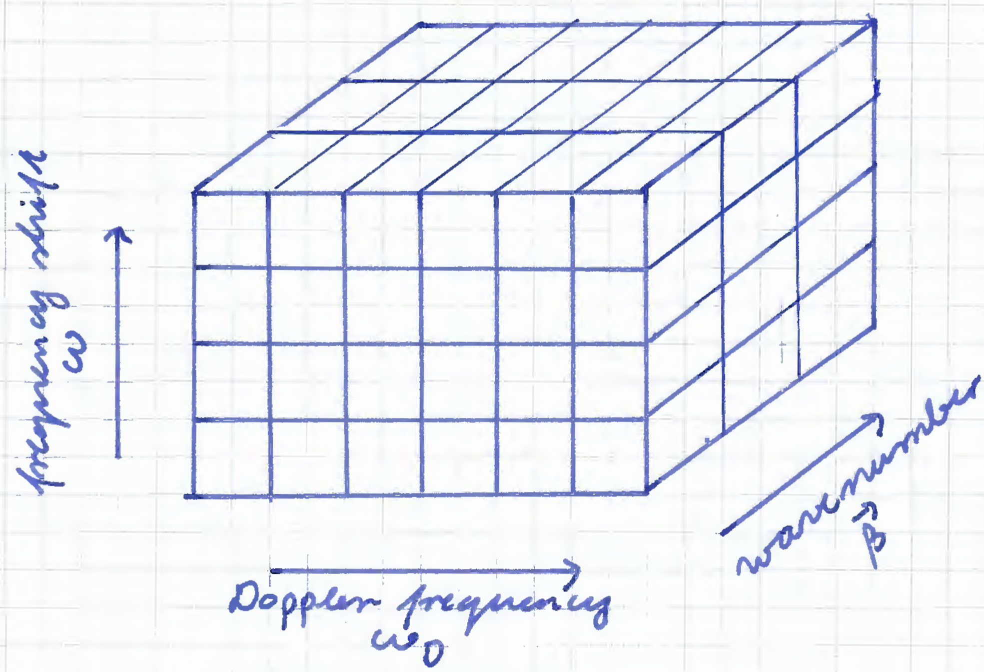
$$\varphi_{n,p,k}(t) - \varphi_p(t) \propto \omega_k (t - pT_p) + \omega_{D,k}\, p T_p + \langle \vec{r}_n , \vec{\beta}_k \rangle \underbrace{- \omega_0 \frac{2}{c_0} r_k + \alpha_k}_{\text{constant}}$$

- challenge: correlated targets (due to illumination by the same transmitter)

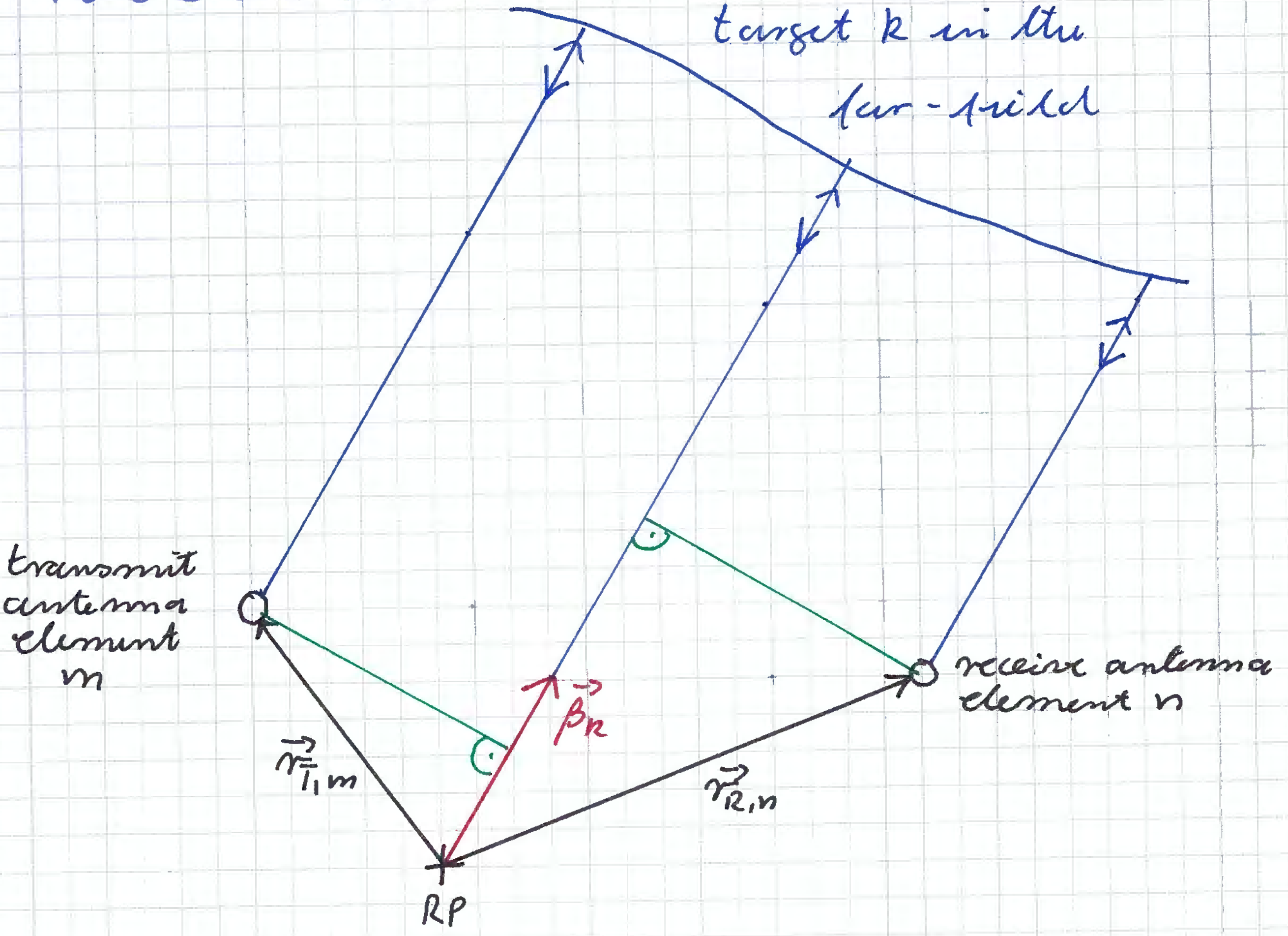
Data Cube



3 D Fourier transform



MIMO Radar



- phase shift:

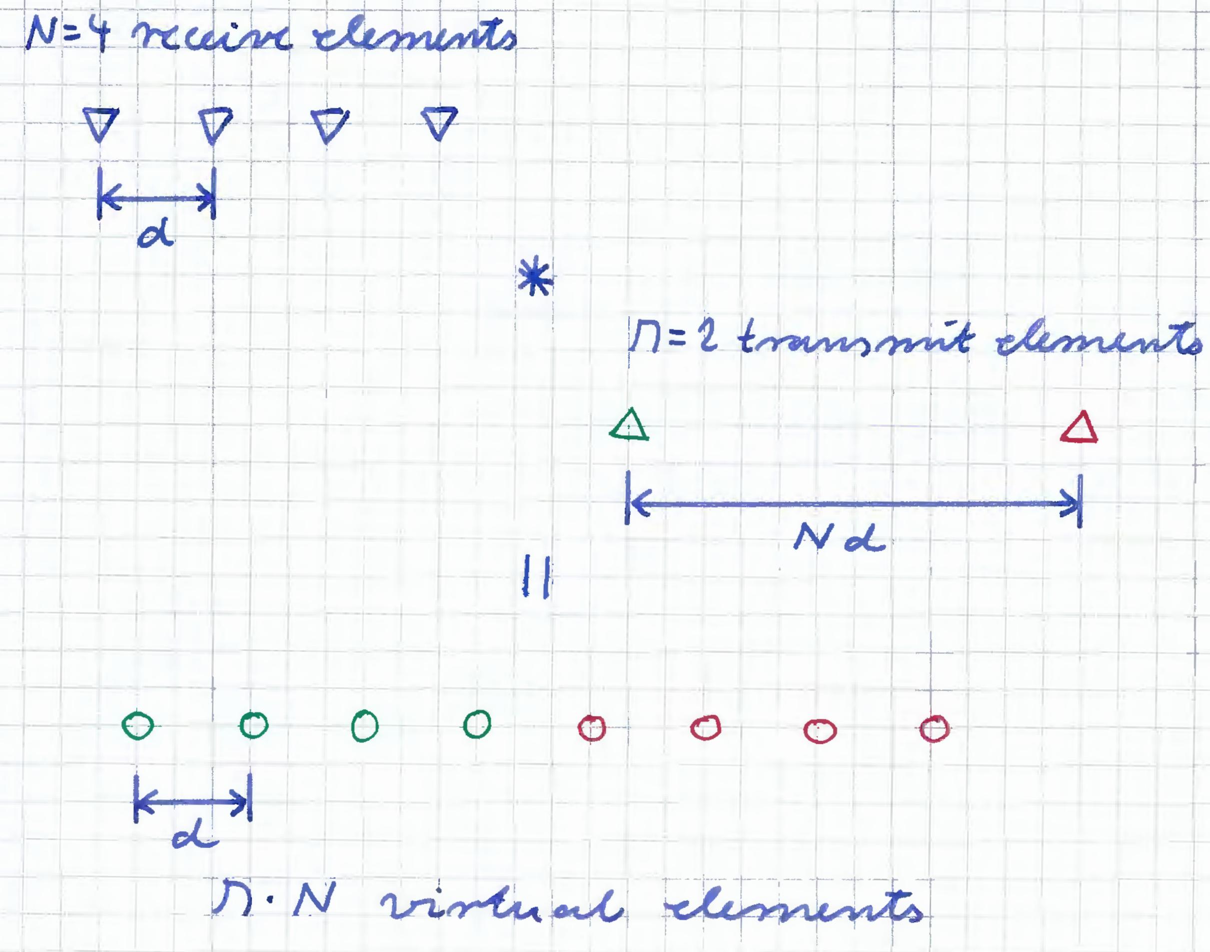
$$\langle \vec{r}_{T,m} , \vec{\beta}_k \rangle + \langle \vec{r}_{R,n} , \vec{\beta}_k \rangle$$

$$= \langle \vec{r}_{T,m} + \vec{r}_{R,n} , \vec{\beta}_k \rangle$$

$\Rightarrow$ virtual antenna element at position $\vec{r}_{T,m} + \vec{r}_{R,n}$

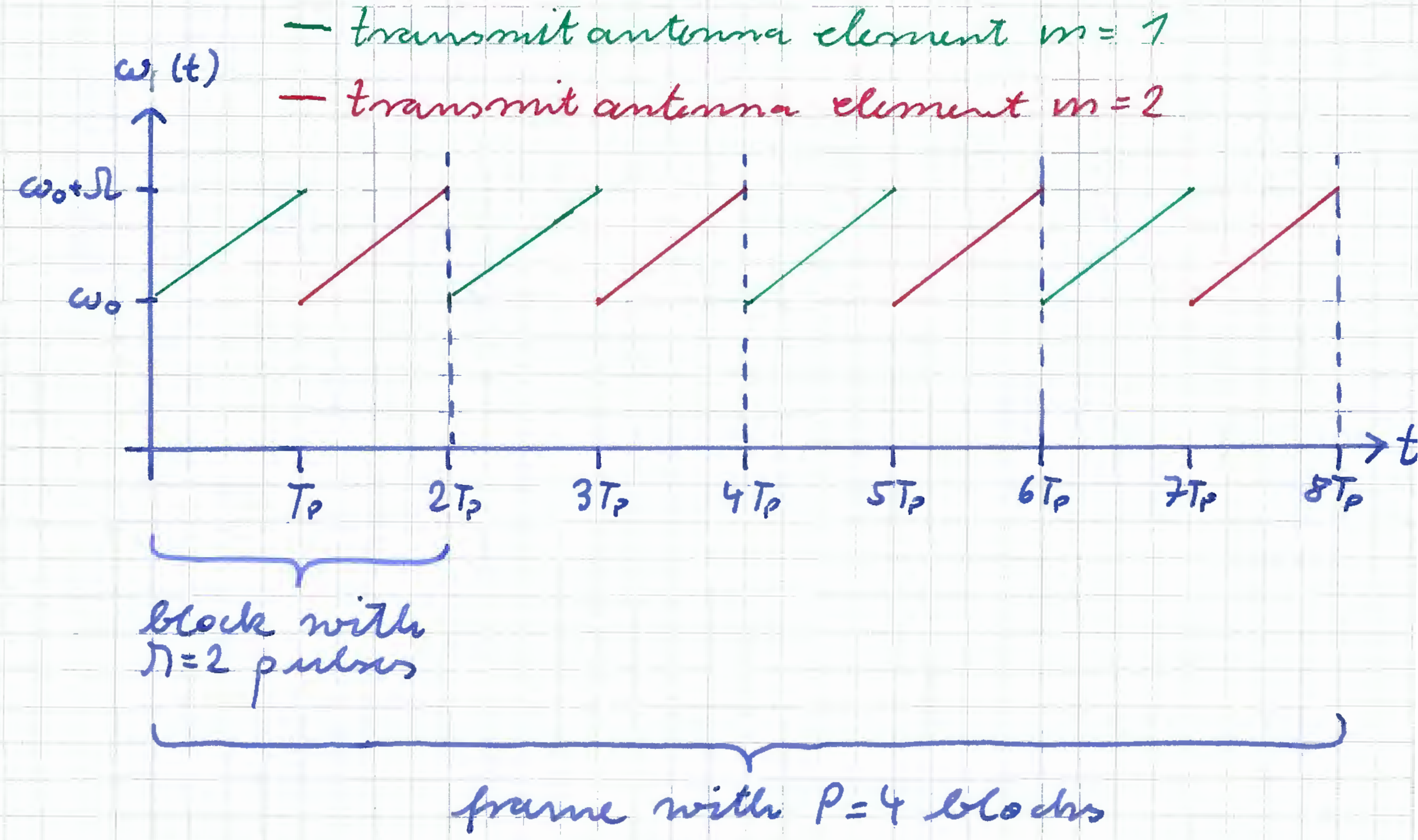
Virtual Array

convolution of the transmit array
and the receive array



Time Division Multiplexing

- signals coming from different transmit antenna elements must be separated
- time division multiplexing: antenna elements transmit at different times



Doppler Correction

- phase of target k in pulse $m+\Pi p$ at receive antenna element n:

$$\Psi_{n,m+\Pi p,k}(t) - \Psi_{m+\Pi p}(t) \approx \omega_k \underbrace{(t - mT_p - \Pi p\, T_p)}_{\ell T}$$

$$+ \omega_{D,k}\, m T_p + \omega_{D,k}\, \Pi p\, T_p$$

$$+ \langle \vec{r}_{T,m} + \vec{r}_{R,n} , \vec{\beta}_k \rangle$$

$$\underbrace{- \omega_0 \frac{2}{c_0} r_k + d_k}_{\text{constant}}$$

- the term $\omega_{D,k}\, m T_p$ has to be compensated prior to spatial processing
- sampling rate in slow time domain reduced by a factor of Π